Preprint of: Kazmer, David O, and Christoph Roser. “Evaluation of Product and Process Design Robustness.” *Research in Engineering Design* 11, no. 1 (1999): 20–30.

EVALUATION OF PRODUCT AND PROCESS DESIGN ROBUSTNESS

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1. ABSTRACT

Critical design decisions are commonly made throughout product development assuming known material and process behavior. However, the final manufactured product properties depend upon the specific tool geometry, material properties, and process dynamics encountered during production. Moreover, slight random variations during manufacture can inadvertently result in inferior or unacceptable product performance and reduced production yields. Stochastic simulations have been developed to estimate the end-use performance distribution prior to the commitment of hard tooling. This paper extends these methods to model the important role of the manufacturing response in process optimization and elimination of defects. Small changes in the manufacturing method can frequently improve the product quality and eliminate small flaws in the product design. Three contributions are made in the domain of net shape manufacturing. First, a definition for robustness is presented. In addition, in the appendix, this definition of robustness is proven to be convex, making it extremely suitable for optimization techniques. Second, a robust design methodology is introduced and contrasted with conventional development methods. Finally, this methodology is applied in the example of the concurrent design of a molded plastic part.

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KEYWORDS
Robust Design, Monte Carlo Analysis, Conceptual Design, Process Optimization, Probabilistic Design, Convex Robustness, Robust Process

2. INTRODUCTION

The synthesis of new concepts is the primary added value activity of design. While design synthesis utilizes deterministic data, the environment surrounding the manufacture and end-use of the design is largely uncontrolled and stochastic. As such, the design robustness largely determines the product's efficiency, reliability, and perceived quality (Ford, 1995). Figure 1 illustrates a segment of a typical product development process – needs assessment, conceptual design, and many other tasks have been omitted for simplicity.

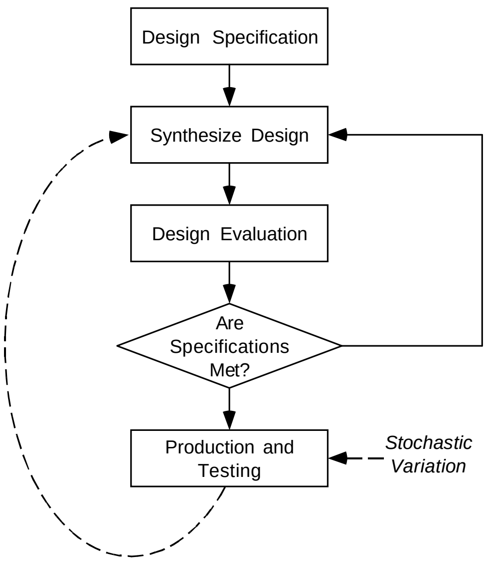

Figure 1: A Common Development Process

In this process, the design specification acts as a contract between the customers and the product development team. During the detail design stage, every effort is spent to ensure that the physical manifestation of the design will meet the required design specifications. Multiple design iterations are commonly evaluated before a 'robust' solution is accepted (Dixon, 1986).

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Unfortunately, evaluation techniques available in the early stages of the product development process do not typically consider the effect of systematic and stochastic variation during production or end-use of the design. This is an important consideration as the design evaluation occurs before production or testing, so the downstream production information is not known and can not be used to influence the design. As such, the resulting product may differ considerably from the idealized design and fail to meet the required design specifications. One or more external design iterations may then be necessary to bring the product to acceptable quality or performance levels as indicated by the dashed feedback loop in Figure 1. As Dacey has indicated, these late design iterations often require costly tooling changes and delay the product launch (Dacey, 1990).

This article describes a concurrent analysis method for explicitly considering the stochastic variation and manufacturing response during production. The goal is to enable the designer to understand and account for the effects of manufacturing and end-use variation. This knowledge could then be applied in the configuration and detailed stages of design to select and tune design parameters and manufacturing processes, thereby delivering robust engineering designs.

3. PRIOR ROBUST DESIGN METHODS

Deterministic optimization techniques have been traditionally applied in the detailed design stage of product development to enhance product performance or reduce unit cost. Examples include shape optimization, wall thickness minimization, and cycle time reduction (Ali, 1994; Burns, 1994; Santoro, 1992). The application of optimization techniques in these instances was possible because well-defined relationships existed between the independent design variables and their performance attributes. These deterministic methods, however, do not consider or predict the impact of stochastic variation in actual material properties, manufacturing processes, or end-use operation.

Two different approaches have been developed to address the issue of variation in manufacturing. Knowing that input variation is unavoidable, Taguchi developed methods of parameter and tolerance design utilizing direct experimental techniques to minimize product variation by maximizing the signal-to-noise ratio. Since the 1970's, Taguchi has shown that robustness can be enhanced in a wide range of applications through use of his Parameter Design Methodology (Taguchi, 1993). These methods have now become commonplace in modern engineering design and manufacturing practice. Wilde (1992) and Sundaresan (1989) have developed other efficient means for maximizing design robustness when computer models exist of the manufacturing process.

Stochastic and probabilistic optimization (Charnes, 1963) is a separate approach which considers the effect of random variation in the assessment and optimization of a design's performance. As with all optimization problems, the approach and formulation are critical components in developing a useful model relating input variation to end use properties. In stochastic optimization, variables are described by distribution functions instead of deterministic constants. The goal is to determine an optimal design that satisfies the required specifications with the highest reliability. Eggert and Mayne (1993) and Lewis and Parkinson (1994) have provided overviews of this research area.

There are two fundamental differences between the proposed methodology and previous design methods. First, previous research requires the design parameter distributions be known *a-priori* to estimate the effect of variation on system robustness. The proposed method takes one step back, examining the core sources of variation and conveying the effects through the manufacturing process to predict the distribution of end-use product properties. Moreover, the proposed methodology also incorporates an estimate of the manufacturing response to improve the product properties during production when faced with instances of significant variation. Once the model has been developed, the robustness of different candidate designs and processing strategies may be evaluated.

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4. QUANTIFYING ROBUSTNESS

Each component in an assembled product must be designed with a set of functional requirements and specifications, some implicitly understood, others explicitly stated. Designs to be produced using net shape manufacturing processes pose two significant challenges for the designer which this method will address. First, processing as well as geometric design and material properties determine the final product properties and performance. Second, there is significant coupling between design, material properties, and processing. For instance, small changes in the specification of a wall thickness for a molded part may result in large swings in the cavity pressure distribution which, in turn, may inadvertently affect the material shrinkage and part dimensions thereby rendering an unacceptable product.

These form-fabrication-function relationships are compounded in technical applications with multiple requirements, subject to process dynamics and limitations that are unknown to the product designer. To overcome these difficulties, improved analysis techniques have been developed to better predict product performance for candidate designs. In theory, more accurate analysis techniques could eliminate the need for costly mold tooling and evaluation iterations. In reality, even the most advanced analyses remain incapable of providing accurate estimates of performance for candidate designs given the effects of uncertain material properties and stochastic process variation. As such, the product development process for net-shape manufacturing processes is forced to utilize iterative evaluations in which steel must be cut with no guarantee that the mold alterations will deliver the desired product performance.

Let us consider another example: what wall thickness should be used to minimize the cost of a die-cast part while ensuring adequate manufacturability and structural performance? The product development team must specify geometric design parameters, material properties, and process conditions. These design decisions influence certain output characteristics such as cooling time, part weight, flow length, and moment of inertia which are of concern to the development team. However, it is the exact state of the net-shape process during a part's manufacture which the development team can not know a-priori (*let alone measure in-situ!*) which will ultimately determine the actual end-use properties of the manufactured product. As such, several design-build-test iterations may be required to achieve the desired performance.

Unexpected variation can result in unsatisfactory product performance, low production yields, and increased product cost. The objective of this design methodology is to enable the creation of robust designs whose manufactured product properties are within desired specifications, even in the presence of uncertain material properties and stochastic process variations. Robustness has been defined in terms similar to the process capability index (*Cp*) which is used in characterized manufacturing process (Boyles, 1991) for “the nominal the better” specification:

$$\Re = \frac{\left((USL - LSL) - 2\left|\bar{\mu} - \tau\right|\right)}{6\sigma}, \text{where:}$$

$$\begin{aligned} USL &\equiv \text{upper functional limit of product requirement} \\ LSL &\equiv \text{lower functional limit of product requirement} \\ \tau &\equiv \text{target of product requirement} \\ \bar{\mu} &\equiv \text{mean of product performance} \\ \sigma &\equiv \text{standard deviation of product performance} \end{aligned}$$

Equation 1

Other types of targets, including one-sided “the smaller/larger the better” specifications have been defined elsewhere and are likewise treatable.

This definition of design robustness indicates the ability of the manufacturing process to deliver products that satisfy the specified product requirement – a robustness equal to one represents product performance at the target level with three standard deviations to the closest specification limit. If a 12σ level of quality is specified to correspond to Motorola 6σ guidelines (Denton, 1991), the design robustness is required to be 2 or higher. The robustness of a design with multiple requirements may be evaluated via the joint probability of feasibility as:

$$P_{total} = \prod_{i=1}^{n} P_i$$

Equation 2

where

$$\Re_i = \frac{-1}{3}\Phi^{-1} \cdot P_i \quad \forall i$$

Equation 3

which is also valid for the relation between $\Re_{total}$ and P_{total}. Combining those two equations gives:

$$\Re = \frac{-1}{3}\Phi^{-1}\left(\frac{1}{2} - \frac{1}{2}\prod_{i=1}^{n}\left(1 - 2\Phi\left(-3\Re_i\right)\right)\right), \text{where:}$$
$$\Re_i \equiv \text{Robustness of i - th performance parameter, eq. (1)}$$
$$\Phi \equiv \text{Normal cumulative density function}$$
$$\Phi^{-1} \equiv \text{Inverse normal cumulative density function}$$
$$n \equiv \text{Number of performance parameters}$$

Equation 4

Thus, design robustness is an aggregate performance measure that includes the consequences of product and tolerance design, process capability, and stochastic variation. There are several beneficial properties of this definition for robustness:

* models multiple design objectives;
* convex behavior allows for global optimization;
* allows for direct inclusion of different kinds of specifications;
* consistent with Taguchi’s concept of tolerance design since it promotes central tendencies with small deviations in product properties, rather than a goal post mentality (Devor, 1992); and,
* consistent with many design axioms to minimize information content since the production yield will tend to decline geometrically as the number of requirements rise (Suh, 1990).

5. METHODOLOGY

The described methodology, presented in Figure 2, explicitly considers stochastic variation in both the design and manufacturing processes – the technique utilizes optimization of the manufacturing process conditions within a stochastic simulation to evaluate the robustness of a candidate design for a stochastic manufacturing process. The output of the methodology is a robust product and process design that does not require iterations during tool commissioning. An overview of the methodology using a Monte Carlo method as the stochastic optimization is given in Figure 1.

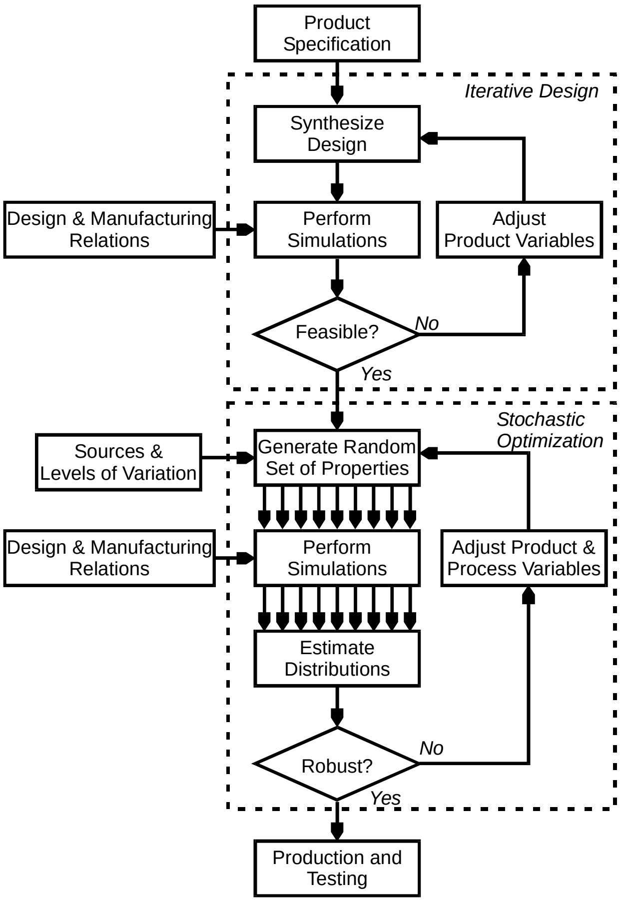

Figure 2: Robust Product & Manufacturing Design Methodology

5.1. Methodology Requirements

To enable evaluation of the design and manufacturing robustness, the following items are required:

* a set of product specifications, indicated by the vector τ;

* a candidate design represented by the design variables, **x**, as well as initial estimates of the manufacturing process described by **y**;
* an estimate of the sources and levels of variation within the design, δ**x**;
* an estimate of the sources and levels of variation within the manufacturing processes, δ**y**; and,
* a set of design to manufacturing relationships, usually implemented as a numerical simulation, to predict the properties of manufactured products from design, material properties, and process dynamics.

The product specifications are a basic element of every design. The Design and manufacturing relations are required to evaluate the product design, and finally the sources and levels of variation for the design and the manufacturing process have to be known in order to generate input values for the stochastic analysis. Each of the requirements listed above is described below in more detail.

5.1.1. Product Specifications $\boldsymbol{\tau}$

Within this methodology the product specifications are assumed to be given. Once a candidate design is synthesized, a random set of design and process conditions is instantiated as consistent with $\{\delta\mathbf{x}, \delta\mathbf{y}\}$. Given this instance of the design, a simulation is performed to estimate the manufactured product's end-use properties, represented by the vector $\boldsymbol{\mu}$. The expected value of each product property, μ_i, will be compared to its specification, τ_i. Therefore, the product specifications are the constraints for the product parameter. There are two types of constraints to be applied in this algorithm: design constraints and process constraints. Design constraints are imposed by the designer on the allowable range of adjustable design variables, **x**. For instance, constraints are typically used to guarantee that a length must be maintained within tolerances or a wall thickness must be less than 5 mm. Process constraints on **y** stem from the physical limitations of the manufacturing processes.

5.1.2. Design and Manufacturing Relations δx and δy

The first step in the design methodology is to identify critical properties in the final product. Process relations are then necessary to link the design variables, **x**, and process variables, **y**, to the end-use product properties, **μ**:

$$\begin{aligned}
&\mu = f(\mathbf{x},\mathbf{y}), \text{where :}\\
&\mu \equiv \text{set of end use properties}\\
&\mathbf{x} \equiv \text{set of design variables}\\
&\mathbf{y} \equiv \text{set of process variables}
\end{aligned}$$

Equation 5

There are many reliable methods for developing functional models, including empirical, analytical, and numerical techniques. Unfortunately, net-shape manufacturing processes are notoriously complex, with highly non-linear interactions between design, material properties, process conditions, and end-use properties. The development of adequate models is a big step – the number of factors and complex interactions between factors make it difficult to predict the resulting product properties. In the absence of available models, one might well profit from a Taguchi-style design of experiments to identify the critical variables and their effects/interactions.

5.1.3. Sources and Levels of Variation

The second step in the design methodology is to identify the root sources of variation and understand the mechanisms of variation in production. Sources and levels of stochastic variation must be assessed to evaluate the robustness of the product design and process capability in the presence of unknown material properties, random process variation, and other factors. Some of the real-life sources of variation that could be considered using this design methodology:

* inconsistencies in material properties, such as batch-to-batch variation;
* effect of unmodeled or unknown material properties;
* systematic errors in process conditions;
* random, time-varying process noise; and
* inaccurate design or end-use assumptions.

All of these factors may vary significantly across a product's development and life cycle. The evaluation methodology requires probabilistic ranges to be applied to each of the root cause variables. In the following application, each of the many design and process variables are assumed to be stochastic and normally distributed with standard deviations. The methodology, however, is not restricted to any unique probability function and may easily be extended to consider arbitrary sources and distributions of variation.

5.2. Iterative Design

The first of the two loops of the presented methodology is similar to the conventional design methodologies (see Figure 1). A set of design variables x is chosen based on previous experience with similar designs, engineering knowledge or other design methodologies. Next development process, this design was then evaluated using simulations, testing, models or other evaluation techniques. Depending if the outcome of the evaluation does meet the specifications the design is accepted, otherwise the design variables where modified and a new set of design variables x was evaluated. This process gives after multiple iterations a optimized setting for the design variables x. In this process, the process variables y receive little or no attention with regards to the product performance, yet they play an extremely important role for the final quality and robustness of the product. Frequently design optimization techniques stop at this point and ignore the statistical aspect of the design. This methodology, however, differs as now the resulting nominal design variables are analyzed and optimized in a probabilistic sense, including the process design variables x **and** the process variables y.

5.3. Stochastic Optimization

There are different ways to evaluate the stochastic behavior of a design. If the functional relation of the product and process variables x and y and the product properties μ are known, then a sensitivity analysis can be performed and the distribution of the product properties can be estimated.

However, usually the functional relationship is either not known at all or very complex, making a sensitivity analysis very difficult. Therefore, other methods are used to evaluate the product design and process from a statistical point of view, for example moment matching methods and Monte Carlo simulations. Compared to moment matching methods, Monte Carlo methods are easy to implement, highly accurate, and enable consideration of arbitrary, complex, and mixed probability distributions. The one predominant disadvantage, of course, is computation time with thousands of function calls being required to estimate robustness. If the function call is a complex numerical simulation, evaluation time can exceed hours or even weeks. For the methodology presented here the Monte carlo method was chosen, however, this methodology is easily adaptable to other techniques like sensitivity analysis and moment matching methods, and the best method depends on the design and the understanding of the system.

The Monte Carlo simulation algorithm requires that multiple instances of random variables are generated for the design and manufacturing variables, $\{\tilde{\mathbf{x}},\tilde{\mathbf{y}}\}$. For instance, a randomized set of stochastic design variables, $\tilde{\mathbf{x}}$, may be generated as:

$$
\begin{aligned}
\tilde{\mathbf{x}} &= N(\bar{\mathbf{x}}, \delta\mathbf{x}) = \Phi^{-1}(\text{random}(1)) \cdot \delta\mathbf{x} + \bar{\mathbf{x}}, \text{where :} \\
\tilde{\mathbf{x}} &\equiv \text{set of design variables with Gaussian distribution} \\
\Phi^{-1} &\equiv \text{Inverse normal cumulative density function} \\
\delta\mathbf{x} &\equiv \text{set of design variable standard deviations} \\
\bar{\mathbf{x}} &\equiv \text{set of design variable expected mean}
\end{aligned}
$$

Equation 6

Then, those multiple instances are evaluated and the distribution of the product parameters is estimated. Depending on those estimated output distributions the robustness of the product is calculated. If the robustness criteria of the product are met then the algorithm stops and the product can be produced and tested, otherwise the product **and** process variables are adjusted. If two design goals are conflicting, τ_i and τ_j, then a set of process conditions will be selected that makes a compromise between the two to maximize the overall utility:

$$\Re = \underset{\tilde{\mathbf{y}}}{\text{maximize}}\left\{\frac{-1}{3}\Phi^{-1}\left(\frac{1}{2}-\frac{1}{2}\prod_{i=1}^{n}\left(1-2\Phi\left(-3\Re_i\right)\right)\right)\right\},$$

$$\text{where}:$$

$$\Re_i = \frac{\left(\left(UFL_i - LFL_i\right) - 2\left|\mu_i - \tau_i\right|\right)}{6\sigma_i}$$

$$\mu = f\left(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}\right)$$

Equation 7

5.4. Output Robustness

Through the described methodology, the product and process robustness is evaluated. A robustness of 1.0 corresponds to the predicted molded part property being centered between the upper and lower specification limits, with a predicted standard deviation of one sixth of the tolerance band – this corresponds to a production yield of 99.3%. For higher values of robustness, the production yield approaches 100%, indicating that the manufacturing process can always be adjusted to compensate for random sources of variation and manufacture products within the required specifications. If a candidate design is not feasible or a manufacturing process is overly inflexible, then the robustness will be considerably less than 1.

In the manufacture of complex net-shape products, such as injection molding of automotive instrument panels, initial production yields of ~95% are often considered acceptable. By utilizing process relationships with wide probabilistic spreads in the evaluation, a predicted robustness near 1.0 indicates that the process flexibility exists to meet the required product specifications and that re-tooling or additional design iterations should not be necessary in production.

If the predicted robustness is significantly lower than 1, rework of the design or consideration of a different manufacturing process may be necessary to increase the robustness of the product. The results of the evaluation will indicate which constraint or objective is causing the loss in the product robustness, suggesting a starting point for the product or process redesign. This may involve changing the gating scheme, varying the thickness, increasing allowable tolerances, or other numerous actions. When corrective actions have been completed, the relative success of the new design may be evaluated. As with all optimization techniques, the designer's experience plays a crucial role in the evaluation and acceptance of a candidate design. If the 'optimal' design is not acceptable, the designer must re-formulate the optimization problem, adjust the relationships between design variables and product performance, and guide the design to a more satisfactory design space.

6. APPLICATION TO DIMENSIONAL DESIGN FOR THE INJECTION MOLDING PROCESS

Tight tolerance and technical molding applications are becoming increasingly common as the injection molding process continues to emerge as the premier vehicle for delivering high quality, value-added products to the marketplace. These applications have increased standards for product capability and quality which challenges the ability of design and process engineers to deliver acceptable molded parts on time and under budget. In fact, several industry managers have testified that "we are starting to see the migration of customers to other manufacturing processes for time-critical applications."

Practitioners are utilizing increasingly sophisticated design analyses and molding processes in an effort to minimize the time and cost required for development of molding applications. In theory, these advanced technologies provide more robust product and tool designs while reducing the sources of manufacturing variation. In reality, the performance and added value of these methods is not always clear. Design and process engineers need to know the comparative gains that can be made by adopting a process before physical implementation.

The described methodology was applied to evaluate the robustness of different product and process designs by comparing standard operating procedures to industry best practices. The results quantify the likely impact of development strategies from which developers can select the strategy with the appropriate cost:benefit characteristics. Altogether, three different ‘best practices’ are investigated for tight tolerance applications:

* a design engineer minimizing the number of critical design specifications on a molded part;
* a tool engineer utilizing constant material shrinkage versus differential shrinkage estimates in mold tooling; and,
* a process engineer re-optimizing the process with material and environment shifts.

With this information, the product and process development team can determine the correct implementation and quality strategy. While these applications of the methodology were developed to provide a valuable example for the plastics industry, it should be clear the described methodology is readily extensible to other types of product designs and process technologies.

6.1. Product Description

The molding application is an electronics housing, shown in Figure 3. The part is molded of CYCOLOY ™C2950 resin, an ABS-PC blend from GE Plastics (Pittsfield, MA). The melt is conveyed into the cavity through a direct, center-sprue gate. The nominal processing conditions for the filling stage consisted of mold and melt temperatures of 70 C and 270 C, and an injection time of 1.5 seconds. A packing pressure of 50 MPa was then maintained for 5 seconds, followed by a twenty second cooling time.

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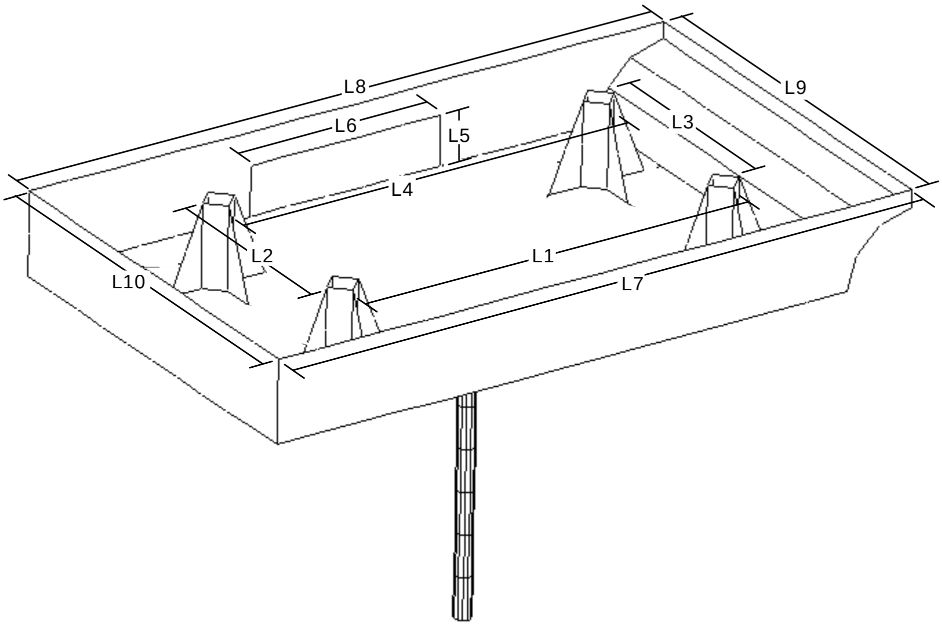

Figure 3: Typical Molded Part and Specified Dimensions

In this application, the design specification includes three critical dimensions for locating and attaching a mating part to the four gussetted bosses shown in Figure 3. In this instance, only the dimensions L1, L2 and L3 are considered critical, therefore the other dimensions will be ignored. For reference, dimension L1 has been specified as 250 ± 0.2 mm while dimensions L2 and L3 have been specified as 100 ± 0.2 mm.

Table 1: Dimensions

Dimension	Nominal Value (mm)	Tolerance (mm)
L1	250	± 0.2
L2	100	± 0.2
L3	100	± 0.2

The specified tolerances are not actually 'tight' but more typical of industry standards. Given this part description, the goal is to quantify the impact of the described best practices.

6.2. Problem Formulation

In this tight tolerance application, the molded part dimensions are the primary measures of performance, μ. To deliver the desired product performance, the development team can adjust the tool dimensions represented by **x**, the design variables. The material behavior is also a design parameter, but will exhibit stochastic variation during production. The manufacturing control variables, **y**, include various temperatures, pressures, and velocities which have a controlled mean but may vary stochastically.

6.2.1. Design and Manufacturing Relations

Computer simulations have been developed which employ physical laws (i.e. the continuity equation, momentum equation, and energy equation) to simulate the machine and plastics behavior (Hieber, 1978). The capabilities of these analyses to predict part dimensions have been well documented (Fox, 1998). As such, a commercial computer aided engineering analysis, Moldflow, was utilized to estimate the molded part dimensions for each instantiated set of design and manufacturing conditions.

6.2.2. Sources and Levels of Variation

Table 2: Sources and Magnitudes of Variation

Sources	Mean	Standard Deviation
Melt Temperature	240 C	5
Mold Temperature	80 C	8
Injection Time	2.0 sec	0.2
Pack Pressure	50 MPa	3
Pack Time	5 sec	0.2 sec
Cooling Time	20 sec	1.0 sec
Polymer Viscosity	250 Pa Sec	10
Polymer Density	1.02 gr/cm3	0.06

The choice and characterization of sources of variation, described in Table 2, was chosen to emulate the range of noise that would be encountered in a typical production scenario of 100,000 parts being produced on four different machines (Kazmer, 1997). For instance, a ± 5°C fluctuation in melt temperature represents the variation in actual melt temperatures across different molding machines and molders. The ± 8°C range of mold temperatures might reflect variation in water flow rates through the tool which have not been specified and therefore vary more between different set ups. Similarly, the levels of injection speed and hold pressure shown in the Table 2 are indicative of the machine to machine variations in barrel, hydraulic, and controller systems. Additional sources of variation were selected to represent natural material variation, typically due to changes in composition or compounding.

6.3. Relation between Number of Dimensions and Robustness

To quantify the impact of this best practice for part design, a series of stochastic simulations were coupled with mold filling simulations using the described methodology. For each instantiated set of material properties and machine parameters, a process simulation was performed to estimate the yield when only L1 is specified, then L1 and L2 being specified, and so on until all ten dimensions were specified as critical. The resulting process yield (as calculated by equations 1 and 2) is shown in Figure 4 as a function of number of critical dimensions.

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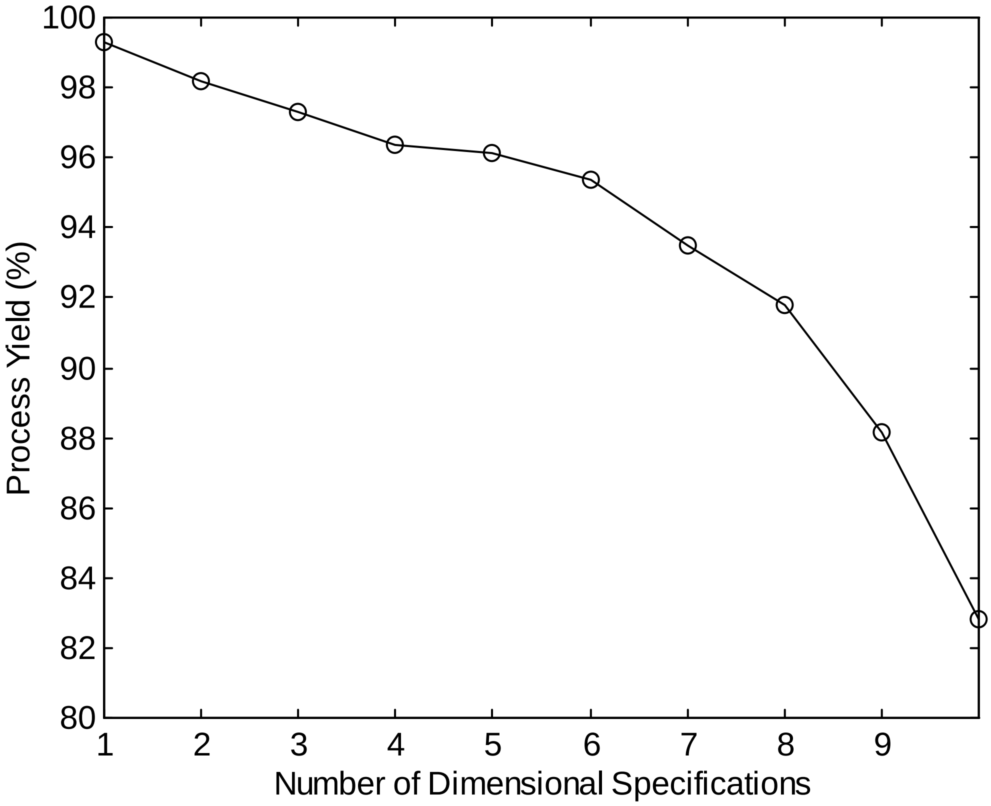


Figure 4: The Impact of Number of Specified Dimensions on Process Capability

Figure 4 shows that the process capability will be less than one regardless of how many dimensions are specified as critical. For instance, the process capability is 0.83 which corresponds to a 99.3% yield when only L1 is specified as critical. This indicates that the standard molding practice of utilizing uniform shrinkage estimates and standard operating procedure is not capable of delivering high yields of tight tolerance molded parts. Some process improvements in mold design or molding practice will be necessary.

As additional dimensions are specified as critical, the process capability quickly degrades. Interestingly, the shape of the curve indicates which dimensions are easier to achieve and maintain. For instance, dimensions L4 and L5 (which are considered non critical and therefore are not analyzed in the example) result in fairly low reductions in the process capability. This is due to the fact that the dimensions are close to the gate, and that the tolerances are fairly large compared to the measured lengths. In contrast, dimension L10 results in a significant drop in the process capability since it is at the end of fill and has a relatively tight tolerance of 200 ± 0.2 mm.

6.4. Results

In the following, the methodology described above is applied in two different ways in order to increase robustness. First, the response is centered between the tolerance limits by changing the tool dimensions to adapt to the different shrinkage. However, this is easy to do if the tool exists only virtual, but if the tool is already cut retooling is rather expensive. the second way this methodology is applied is by adjusting the process parameter with respect to different machines in order to reduce the variation. This is possible with little cost even after the whole production is already set up.

6.4.1. Centering the Responses by Retooling

Given a set of specified part dimensions and tolerances, tight tolerance mold design guidelines are used to attain and maintain the desired part attributes. These tooling guidelines commonly include: 1) utilize a uniform wall thickness across the part to reduce differential shrinkage, 2) build adequate stiffness into the mold base to reduce mold deflection, and 3) use multiple mold interlocks to reduce dimensional play in the parting plane.

The fundamental issue in achieving tight tolerances is the control of non-uniform shrinkage caused by temperature, pressure, and orientation distributions across the part. Figure 5 plots the pressure contours in the electronics housing at the end of the filling stage. Even though the sprue gate has been placed in the center of the cavity, the slightly asymmetric part topology causes significant variation in the cavity pressure distribution. These pressure differentials will continue during the packing stage, resulting in varying volumetric shrinkage during the melt solidification.

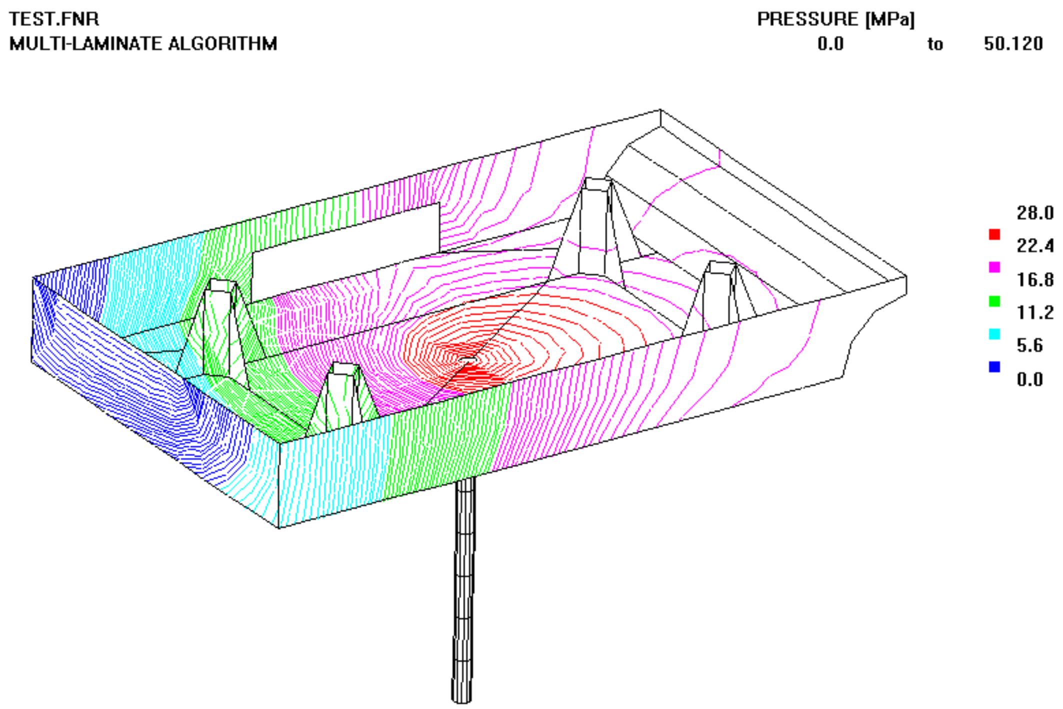


Figure 5: Deterministic Cavity Pressure Distribution During Manufacture

Standard tool design practice is to utilize a nominal estimate for material shrinkage. This estimate is often provided by the material supplier with instructions for the tool designer to cut ‘steel-safe,’ such that more metal can be removed should mold changes be necessary to obtain acceptable dimensions. Using the described methodology, the fundamental sources of variation in the material properties and machine parameters were modeled using Monte Carlo techniques and computer simulation to estimate the resulting distribution of part dimensions. The results of five hundred iterations are shown in Figure 6 for standard tool design practices.

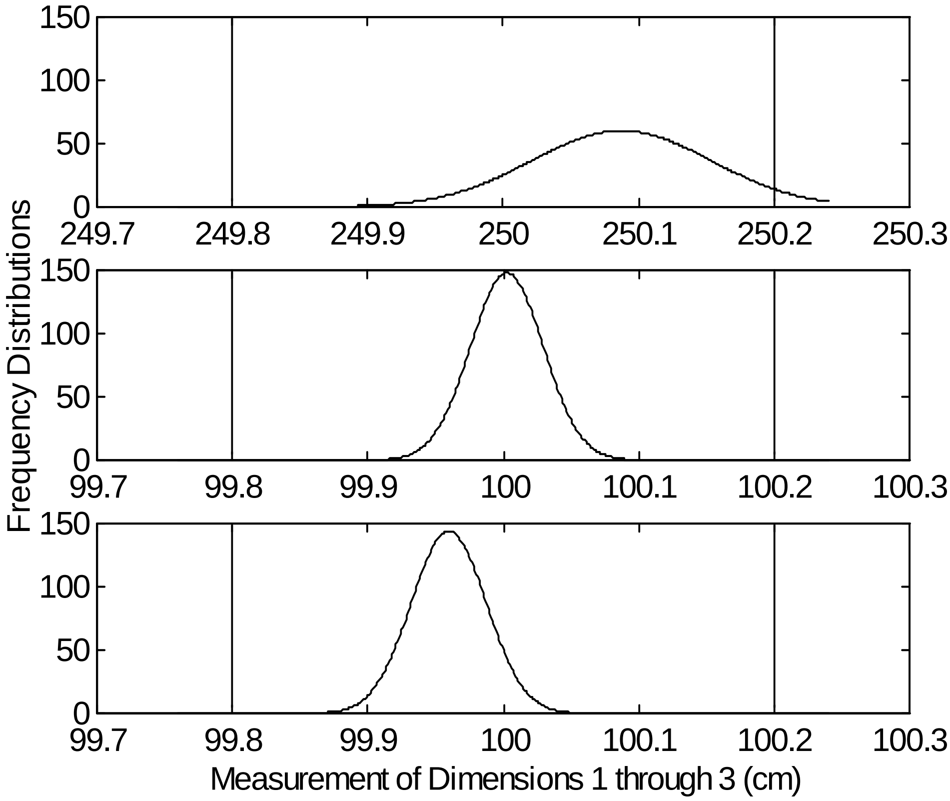


Figure 6: Distribution of Part Dimensions with Standard Tool Design

In this application, the tool designer utilized a correct estimate of 0.55% for the material shrinkage. As Figure 6 shows, however, the non-uniform cavity pressure distributions generated differential material shrinkage and part dimensions. As such, 96% of the molded parts will be acceptable, corresponding to a process capability of 0.62. It should be noted that changing the process conditions will not improve the part properties. Moreover, any error in the uniform shrinkage estimate would only reduce the process capability. For instance, a reduction in pack pressure or the shrinkage estimate might improve dimension one but would worsen dimension three. To improve the yield, additional tool iterations would be necessary to individually tune in the mold steel dimensions.

Computer simulations have become fairly accurate in predicting material shrinkage and the resulting part dimensions. As such, a best practice for tight tolerance molding has been recently proposed: utilize differential shrinkage estimates for calculation of mold steel dimensions. With this methodology, steel dimensions in areas of high pressure near the gate would be cut for less shrinkage than those areas at the end of fill. Knowing the deterministic pressure distribution across the cavity, this example utilized differential shrinkage estimates of 0.55% for L_1, 0.5% for L_2, and 0.60% for L_3. Additional analysis were then performed resulting in the property distributions shown in Figure 7.

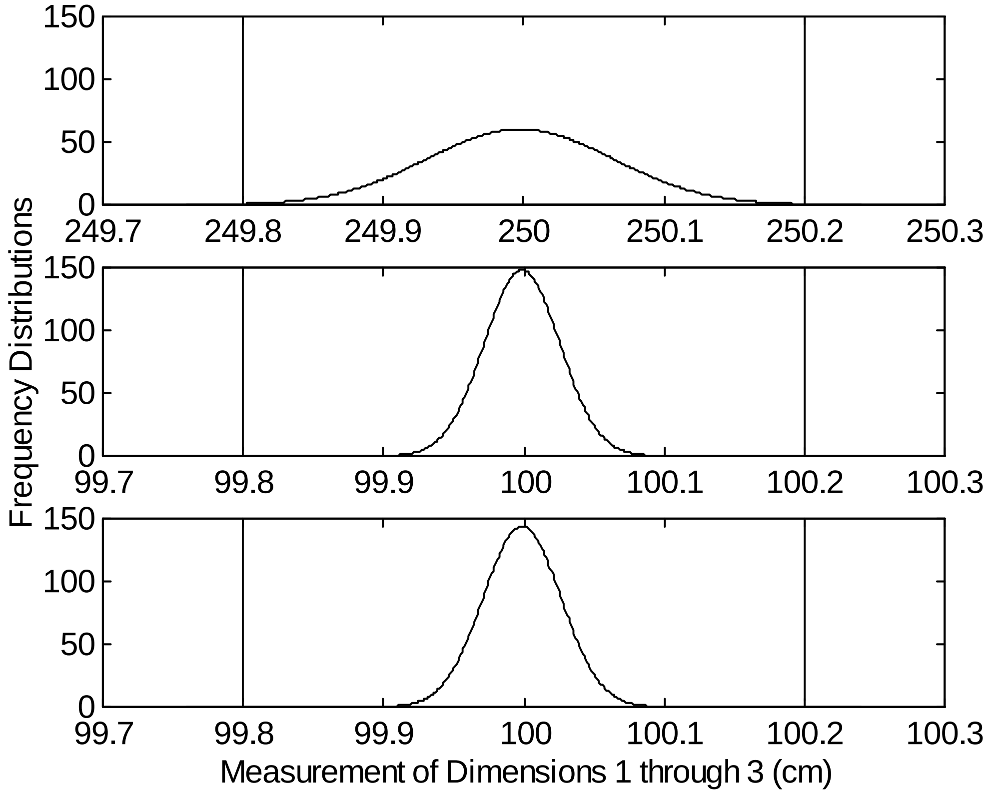


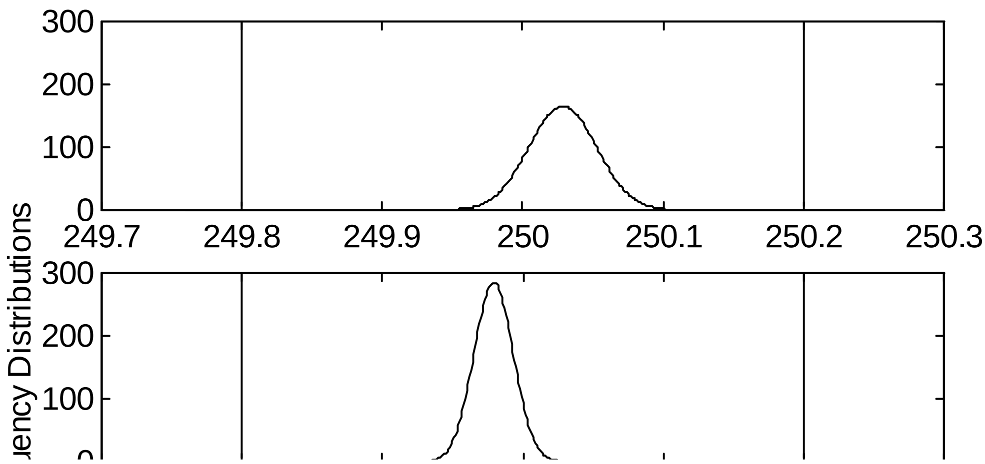
Figure 7: Distribution of Part Dimensions with Best Practice Tool Design

While dimensional variation has not been reduced, the resulting part dimensions are centered between the specification limits. The yield has increased to 99.86%, corresponding to a process capability of approximately 1.0. It should also be noted that slight errors in the shrinkage estimates would continue to provide better part properties than the standard practice of using uniform shrinkage estimates. Thus, tooling best practice would be to use conservative differential shrinkage estimates, which at least qualitatively reflect the expected behavior of the material shrinkage. However, if the material and shrinkage behavior is not well understood, the tool designer should resort to standard industry practice and utilize uniform and conservative shrinkage estimates.

6.4.2. Reduction in Variation by Process Parameter adjusting

The previous design utilized constant process conditions across all the runs. A best practice approach, becoming more common in industry, is to qualify the process for a given mold geometry on a specific machine with a specific lot of material – to continually optimize the process to achieve higher manufacturing yields. While there is stochastic variation between molding machines and material lots, the variation within a batch of parts is greatly reduced.

Using the described methodology, the process conditions were re-optimized to maximize the total yield for each instantiated set of material behavior and machine parameters. *Figure 8* plots the distribution of part dimensions for a tool designed using uniform shrinkage estimates.



Preprint of: Kazmer, David O, and Christoph Roser. “Evaluation of Product and Process Design Robustness.” *Research in Engineering Design* 11, no. 1 (1999): 20–30.

Figure 8: Distribution of Part Dimensions with Best Practice Molding

While the dimensions are not precisely centered with respect to the tolerance limits, the standard deviation of the dimensions has been reduced along with the range of dimensions. As such, all three dimensions have a distance of several standard deviations to the closest quality boundary. The robustness of the molding process has been increased to 1.9, corresponding to a yield of approximately 100%, compared to 99.86% for the centered responses due to the change in tool design. This is a considerable improvement in design robustness, especially considering that no additional investment in technology or process capability is required, just a change in the operation of the molding machine.

7. CONCLUSIONS

A methodology has been described for evaluation of design/manufacturing robustness. This methodology is unique in that it is able to model both the fundamental sources of variation in the manufacturing process as well as the likely response of the manufacturing engineer to that variation. In this way, the robustness of the design and manufacturing pair are evaluated simultaneously. As such, the methodology provides a platform from which various design and manufacturing technologies and practices can be evaluated.

This methodology was applied to evaluate best practices for an injection molded, tight tolerance application. The results will vary for every molding application with its unique set of product specifications, mold geometry, and material properties. However, results indicate that control of manufacturing variation provided greater impact than improvements in product or tool design. However, Motorola ‘6 Sigma’ quality levels, corresponding to a robustness of 2, are unlikely to be achieved without a combination of best practices throughout all stage of product design, tool design, and manufacturing. While this example focused on dimensional control for injection molded parts, the methodology can be directly extended to more complex designs, other production processes, and other types of performance specifications.

8. APPENDIX: PROOF OF CONVEX BEHAVIOR

For a multivariate function like Equation 4, a common method for proof of convexity is to compute the Hessian of $\Re(\Re_j)$ and show that it is positive definite for all $\Re_i$ (Hildebrand, 1962). The Hessian of Equation 4, however, is not a well-formed equation due to the characteristics of the normal probability density functions. As such, this definition for robustness can be shown to be convex by proving that the function is convex on every one-dimensional subset of its range. The proof is begun by defining the probability, *p*, an estimate of the probability that a manufactured product will be acceptable across *n* quality requirements:

$$p = \prod_{i=1}^{n} \left(1 - 2\Phi\left(-3\Re_i\right)\right)$$

Equation 8

Since each of the $\Re_i$ are independent, we can define a k_j for each $\Re_j$,

$$k_j = \prod_{\substack{i=1 \\ i \neq j}}^{n} \left(1 - 2\Phi\left(-3\Re_i\right)\right),$$

Equation 9

such that:

$$\Re = \frac{-1}{3}\Phi^{-1}\left(\frac{1}{2} - \frac{1}{2}p\right) = \frac{-1}{3}\Phi^{-1}\left(\frac{1}{2} - \frac{1}{2}k_j\left(1 - 2\Phi\left(-3\Re_j\right)\right)\right).$$

Equation 10

Equation 4and Equation 10 are identical, though Equation 10 is now in the form of a one-dimensional subset of Equation 4 where the impact of the other quality attributes are included within k_j. Now it must be shown that $\Re$ is convex along every *j*-th dimension. By definition (Schneider, 1993), a function $g : X \to (-\infty, +\infty)$ is convex if for all *x,y* contained in *X* and all λ contained from (0,1):

$$g\left(\lambda \cdot x + (1-\lambda) \cdot y\right) \le \lambda \cdot g(x) + (1-\lambda) \cdot g\left(y\right)$$

Equation 11

Providing our definition for robustness as $g(\cdot)$,

$$\frac{-1}{3}\Phi^{-1}\left(\frac{1}{2} - \frac{k}{2}\left(1 - 2\Phi\left(-3\left(\lambda x + (1-\lambda)y\right)\right)\right)\right) \le \lambda\frac{-1}{3}\Phi^{-1}\left(\frac{1}{2} - \frac{k}{2}\left(1 - 2\Phi(-3x)\right)\right) + (1-\lambda)\frac{-1}{3}\Phi^{-1}\left(\frac{1}{2} - \frac{k}{2}\left(1 - 2\Phi\left(-3y\right)\right)\right)$$

Equation 12

Rearranging and substituting the definition of the normal cumulative distribution function, Φ:

$$\Phi^{-1}\left(\frac{1}{2} - \frac{k}{2}\left(1 - 2\left(\frac{1}{2} + \frac{erf\left(\frac{-3\left(\lambda x + (1-\lambda)y\right)}{\sqrt{2}}\right)}{2}\right)\right)\right) \ge \lambda\Phi^{-1}\left(\frac{1}{2} - \frac{k}{2}\left(1 - 2\left(\frac{1}{2} + \frac{erf\left(\frac{-3x}{\sqrt{2}}\right)}{2}\right)\right)\right) + (1-\lambda)\Phi^{-1}\left(\frac{1}{2} - \frac{k}{2}\left(1 - 2\left(\frac{1}{2} + \frac{erf\left(\frac{-3y}{\sqrt{2}}\right)}{2}\right)\right)\right)$$

Equation 13

Simplifying leads to:

$$\Phi^{-1}\left(\frac{1}{2} + \frac{k}{2}erf\left(\frac{-3\left(\lambda x + (1-\lambda)y\right)}{\sqrt{2}}\right)\right) \ge \lambda\Phi^{-1}\left(\frac{1}{2} + \frac{k}{2}erf\left(\frac{-3x}{\sqrt{2}}\right)\right) + (1-\lambda)\Phi^{-1}\left(\frac{1}{2} + \frac{k}{2}erf\left(\frac{-3y}{\sqrt{2}}\right)\right).$$

Equation 14

Since Φ^{-1}, the inverse normal cumulative distribution function, and *erf*, the gaussian error function, are continuous and strictly monotonic, the above statement is true for all k if the equality holds for k contained at the extrema (Hardy, 1978). In Equation 9, it is critical to note that k represents a real probability and will be enclosed in [0,1]. For $k=0$, Equation 14 simplifies to:

$$\Phi^{-1}\left(\frac{1}{2}\right) \ge \lambda\Phi^{-1}\left(\frac{1}{2}\right) + (1-\lambda)\Phi^{-1}\left(\frac{1}{2}\right),$$

Equation 15

which is evaluated as the trivial case:

$$0 = \lambda 0 + (1-\lambda)0 \text{ for all } \lambda \in (0,1).$$

Equation 16

For $k=1$, the operand of Φ^{-1} is exactly the definition of the normal cumulative distribution function which transforms Equation 14 to:

$$\Phi^{-1}(\Phi(-3(\lambda x + (1-\lambda)y))) \geq \lambda\Phi^{-1}(\Phi(-3x)) + (1-\lambda)\Phi^{-1}(\Phi(-3y)),$$

Equation 17

or,

$$\lambda x + (1-\lambda)y = \lambda x + (1-\lambda)y, \text{ for all } \lambda \in (0,1)$$

Equation 18

Therefore, the system is convex if the distributions are assumed to be normal. For non normal distributions additional proofs are required.

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