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DEFECT COST ANALYSIS

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Abstract

The defect cost analysis investigates the various contributions of conflicting quality objectives towards the total cost, using a probabilistic approach estimating the total effect of defects on product quality cost. Two cost drivers occur in every design, material cost and processing cost. The additional cost due to defect parts is usually seen only as one cost driver, however, it can be broken down towards the cost created by each defect type, based on the likelihood of the occurrence of those defects. By reducing those defect-related costs, a more robust design is achieved.

Introduction

Engineering design is a complex and sophisticated task in order to create a successful product. One major element of a successful product is to reduce the cost of the product while satisfying the specifications. Specifications are the requirements towards the design responses to satisfy the user requirements. This is done by constraining the design responses. Here the development team has to achieve a trade-off between the satisfaction of the constraints and the cost minimization. However, the constraint satisfaction and the total cost of the design are coupled, because increased quality usually creates increased cost, but also reduces the likelihood of defects. The demonstrated methodology will aid the development team in understanding this coupling in order to minimize the total cost of the product by decomposing the total cost and assigning the cost towards the different cost drivers. Then the development team can analyze what causes the cost of the product and thus improve the design towards a more economic design. This enables the development team to improve the design focused on the actual cost drivers, avoiding the improvement of design responses which do not contribute towards the total cost. However, it has to be taken under consideration that a design performance is driven by other reasons than cost. This value analysis is not discussed in this paper.

System Overview

The engineering design model consists of input parameters, which are determined by the product development team, and output responses, which are a function of the input parameters. Furthermore a design is subject to noise, distorting the input parameters and the resulting output responses. Figure 1 visualizes the system as utilized in this methodology.

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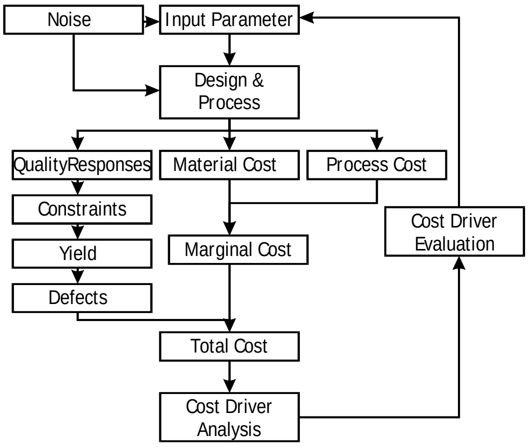

Figure 1: System Overview

The input parameters x_i are applied to the design and the processing of the design, which in turn will determine the output responses y_j. However, as the input parameters of the design and process are subject to noise the output responses will exhibit variation. Therefore, both the input parameters and the output responses are no crisp values but rather a probabilistic distribution. In the following, this distribution is denominated as ϕ. An example for a normal distributed input parameter is given in [1], where the additional information of the mean and the standard deviation is required.

$$\phi_{x_i} = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot e^{\left(\frac{-(x_i - \mu)^2}{2\sigma^2}\right)} \qquad [1]$$

The distribution ϕ of the input factors x_i will determine the distribution ϕ of the responses y_j. It is assumed that the distributions of the constrained responses y_j are independent of each other.

$$\phi_{y_j} = f(\phi_{x_1}, \phi_{x_2}, \phi_{x_3}, \ldots \phi_{xo},)) \qquad [2]$$

In this case, the output responses are divided into three groups, which are part of every design. One response is the utilized material cost C_M, including all materials and standard components needed to create a product, including waste material like sheet metal cut offs or injection molding runner material. It does not include secondary supplies like maintenance material for the production process. These costs are included in the second group of processing cost C_P, which also includes machine cost, man-hour cost, amortized tooling, etc. The third and last group contains the cost due to inadequate quality, in which the resulting part properties are compared with the correlated constraints. Examples for this group are the weight, the strength, chemical resistance, and so on. These responses can be constrained in three different ways such that the response has to be below, above or in between a given limit, where *LSL* is the lower specification limit and *USL* is the upper specification limit.

$$\begin{aligned} &LSL_j < y_j \\ &y_j < USL_j \\ &LSL_j < y_j < USL_j \end{aligned} \qquad [3]$$

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Marginal Part Cost

The marginal part cost C_{MP} is simply the sum of the material cost and the process cost. It is the cost required to produce one additional quantity of the design.

$$C_{MP} = C_M + C_P \quad [4]$$

Yield

In this methodology, a good part is defined as a part satisfying all specifications, and a bad part is defined as violating one or more specifications. Due to the stochastic nature of the input variables and the processing, the same input parameter values will not always result in identical output response values but rather a distribution. Therefore, there is a certain possibility of violating a constraint. The probability density function (pdf) of a normal distributed example is given in Figure 2, where the response is bounded by two constraints, with the likelihood of defect parts equal to the shaded areas of the curve outside of the limits and the likelihood of good parts equal to the non shaded area of the curve between the limits. The probability P of satisfying one constraint if the response y_i is calculated by integrating the response distribution between the correlated *LSL* and the *USL*.

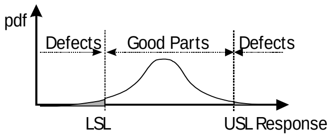

Figure 2: Distributed Response

In case the constraint is one sided then the other side is set to -∞ for the *LSL* or to +∞ for the *USL*.[a] The probability can be evaluated from the evaluation of constraint satisfaction.

$$P_i = \int_{LSL_i}^{USL_i} \phi_{y_i}\, dy_i \quad [5]$$

This integration will always yield a value between zero and one inclusively if the distribution ϕ of y_i is a valid probability density function. The joint probability of success for meeting multiple quality requirements, P_{Joint}, is calculated by multiplying the probabilities P of success of each response y for all n constrained responses, assuming the independence of the responses y.[b] P_{Joint} represents the percentage yield of acceptable parts.

$$P_{Joint} = \prod_{i=1}^{n} P_i = \prod_{i=1}^{n} \int_{LSL_i}^{USL_i} \phi_{y_i}\, dy_i \quad [6]$$

[a] Devore, J.L. (1995) Probability and Statistics for Engineering and the Sciences. Wadsworth.

[b] Papoulis, A. (1991) Probability, Random Variables, and Stochastic Processes. McGraw-Hill.

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Total Cost

After calculating the marginal part cost C_{MP} and the total yield P_{Joint} of the product, the total cost C_T can be calculated. The marginal part cost occurs for every produced part, no matter if the part is defect or acceptable. However, every defect part is rejected, therefore the total cost of the production of one good part also has to include the appropriate cost for the production of the defect parts. Therefore, the total average part cost is calculated as the marginal part cost divided by the yield.

$$C_T = \frac{C_{MP}}{P_{Joint}} = \frac{C_M + C_P}{\prod_{i=1}^{n} \int_{LSL_i}^{USL_i} \phi_{y_i}\, dy_i} \qquad [7]$$

Cost Driver Analysis

So far, the demonstrated procedure is - although not standard in the industry - rather well known in the research community. However, this paper will now reverse analyze the total cost C_T of the product in order to determine the cost driver in the product. One possible approach would be to determine the cost of all defects and compare it with the cost of all defects excluding the investigated defect. This is similar to the cost change if the probability of constraint satisfaction for the investigated response would be perfect with no change to the other responses. Two cost drivers are already known, the material cost C_M and the process cost C_P. The defect cost is then the remainder towards the total part cost. This cost due to the defects C_D can be calculated by subtracting the marginal part cost C_{MP} from the total cost C_T.

$$C_D = C_T - C_{MP} \qquad [8]$$

Now it is possible to compare the defect cost of all defects with the defect cost excluding the analyzed constraint *j*. The defect cost excluding constraint *j*, $C_{De\neg j}$, is calculated similar to the defect cost by ignoring the performance requirement y_j.

$$C_{De\neg j} = C_{MP} \cdot \left[\frac{1}{\prod_{i=1, i\neq j}^{n} \int_{LSL_i}^{USL_i} \phi_{y_i}\, dy_i} - 1 \right] \qquad [9]$$

As $C_{De\neg j} \leq C_D$ the decrease in cost C_{Dej} for a optimal constraint satisfaction $P_j = 1$ would be the difference between the defect cost C_D and the defect cost excluding *j* $C_{De\neg j}$.

$$C_{Dej} = C_D - C_{De\neg j} \qquad [10]$$

While this approach is mathematically justified, it has one flaw reducing its understandability and easy to use. As the joint probability of success, P_{Joint} and the resulting total cost depend on all responses, there exists interaction between C_D and C_{Dej}, i.e. a part may be rejected due to multiple defects. Therefore, the sum of all C_{Dej} may exceed the defect cost C_D.

$$C_D \leq \sum_{j=1}^{n} C_{Dej} \qquad [11]$$

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Although the methodology is mathematically correct, it may lead to confusion if it is expected that the sum of all single defect costs equal the total defect cost. Similarly, a defect cost can be measured by calculating the expected defect cost if all other qualities are assumed perfect, $P_i = 1$, $i \neq j$, and only the defect probability of response j is taken under consideration.

$$C_{Doj} = C_D - C_{MP} \cdot \left[\frac{1}{\int_{LSL_i}^{USL_i} \phi_{yj}\, dyj} - 1 \right] \quad [12]$$

For the same reasons mentioned above, the sum of all single defect costs, C_{Doj}, may be less than the total defect cost C_D.

$$C_D \geq \sum_{j=1}^{n} C_{Doj} \quad [13]$$

Therefore, the presented methodology of defect cost analysis will evaluate the ratio of a single defect probability with the sum of all defect probabilities. This is done by dividing the probability of failure of one response *i* by the sum of all probabilities of failure, which yields the effect *E* of the response *i* on the total cost, where E_j may range from 0 to 1.

$$Ej = \frac{1 - P_j}{\sum_{i=1}^{n} (1 - P_i)} \quad [14]$$

This percentage effect E_i of each constraint violation on the cost due to the yield is then simply multiplied with the cost due to the yield C_Y to get the cost due to the violation of a single constraint C_i.

$$C_j = C_Y \cdot E_j = C_Y \cdot \frac{1 - P_j}{\sum_{i=1}^{n} (1 - P_i)} \quad [15]$$

The cost effect of each defect and quality specification can be measured monetary. In addition, the sum of all single defect costs equals the total defect cost.

$$C_D = \sum_{i=1}^{n} C_i \quad [16]$$

If it is assumed that all quality responses are optimized towards a P_i of one as shown in [15] by calculating C_j, then the C_{Dej} will overestimate the defect cost, whereas the C_{Doj} will underestimate the defect cost.

Cost Driver Evaluation

As stated in [2], the input variables and the output responses are related. Therefore if the effect of the output responses y are known, it is possible to determine which response is the least satisfying, i.e. the most expensive response. Then it is possible to reverse the relation between the input variables and the response to evaluate the input variables, which can improve the selected response. The goal is to adjust those input variables to minimize the total cost of the product.

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Example

The methodology will be demonstrated on an injection molded part as shown in Figure 3.

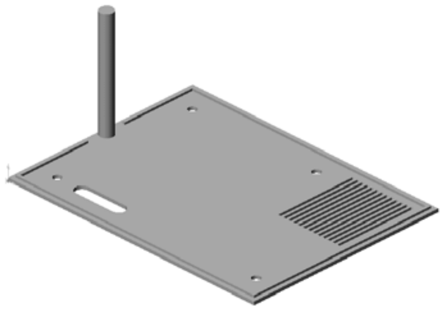

Figure 3: Example Part

This part consists of a flat plane with slots and holes, a small protrusion and a runner system. To simplify the relations between the input parameters and the output responses it is assumed that the part shows the behavior of a flat plate of similar dimensions. The investigated input parameters listed below include a material parameter, a design parameter and two process parameters:

- Molecular Weight
- Melt Temperature
- Injection Time
- Thickness

In order to estimate the quality distribution, a standard deviation was estimated for each input from the knowledge of the process characteristics and applied to the input parameter. The underlying relations between the inputs and the responses are known from different models and simulations, including structural analysis, material prediction, and process simulation. These models also included noise and uncertainties as normal distributions were applied to all model parameters and other input parameters not listed above. To achieve distributed output responses these models and simulations where run repeatedly utilizing design of experiments, with the input variables and model parameters distributed according to the probabilistic density functions of the input parameters. The distribution of the responses was estimated as normal distribution based on the sample data, where m is the number of runs per experiment, l is the numerator for the different runs and i is the numerator for the responses.

$$\mu_{il} = \frac{\sum_{k=1}^{m} y_{ikl}}{m} \quad \forall i,l$$
$$\sigma_{il} = \frac{\sum_{k=1}^{m} \left(y_{ikl} - \mu_{il}\right)^2}{m-1} \quad \forall i,l \qquad [17]$$

The performance specifications included moldability, cost and design responses.

- Deflection

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- Material Cost
- Process Cost
- Cycle Time
- Shear Rate
- Maximum Injection Pressure

Due to the computation time required to calculate the quality responses, a second order response surface was fitted through the data points based on a central composite design of experiments, predicting the mean and the deviation of the output responses.[c] Using this response distribution, the probability of success for one response P_i was determined for each response and subsequent the joint probability of satisfying all responses P_{Joint} was calculated according to [5] and [6].

Figure 4 shows the relations between the input parameters and the yield. It can be seen that the injection time and the thickness have the most significant effect on the yield, as the total yield varies between 0 and 1, whereas the melt temperature and the molecular weight have very little effect on the yield, which is almost constant. Note that the figure shows the relations at the midpoint of the design space. The relations change if the graphs are plotted at another point in the design space due to interactions.

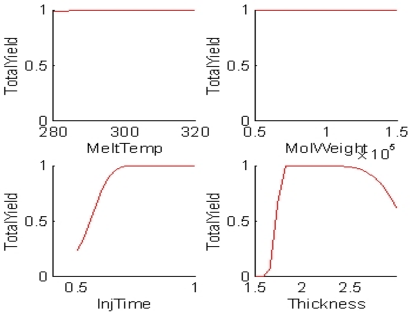

Figure 4: Yield

Figure 5 shows the relations between the input parameters and the marginal cost, where the centerline is the expected mean and the dashed lines on both sides represent three standard deviations from the mean. The marginal cost was created based on the volume of the part, its production time and the required machine size based on the maximum pressure. As there is large variation in the marginal cost, the mean values are flat and only the thickness of the part shows an effect due to the increase in material consumption and cooling time.

[c] Schmidt, S.R., et al. (1994) Understanding Industrial Designed Experiments. Air Academy Press, Colorado Springs, Colorado

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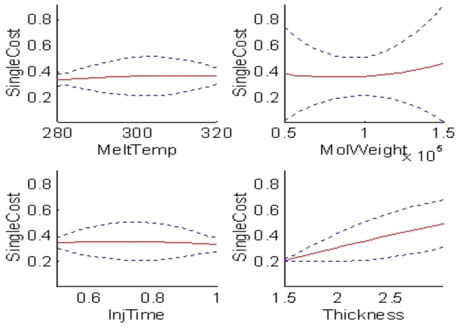

Figure 5: Marginal Cost

Using [7] the total cost is derived based on the marginal part cost and the yield as shown in Figure 6. It can be seen clearly that the total cost rises if the yield goes down. This graph can be derived from Figure 4 and Figure 5. Note that the total cost increases dramatically if the yield becomes very low.

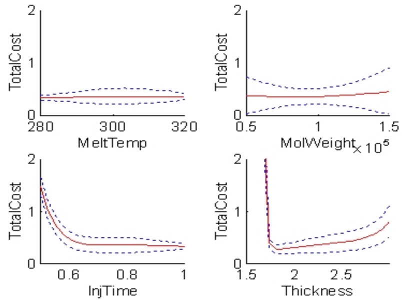

Figure 6: Total Cost

Using the information from above it is possible to estimate the effect of the various cost drivers towards the total cost using [8] and [15]. This effect is shown in a bar graph in Figure 7 for a sub optimal point in the design space with the following values:

- Melt Temperature: 294°
- Molecular Weight: 100,000 mers
- Injection Time: 0.6 s
- Thickness: 1.7mm

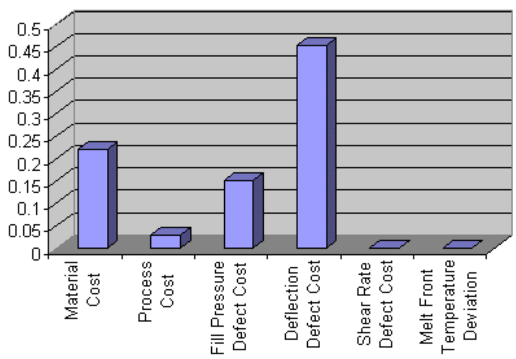

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Figure 7: Cost Driver

The total cost of producing one part for the given design space point is $0.85. The cost drivers for this given design point are

- Material Cost: $0.22
- Process Cost: $0.03
- Fill Pressure: $0.15
- Deflection: $0.45
- Shear Rate: $0.00
- Melt Front Temperature: $0.00

It can be seen that in this case the main cost driver are deflection defects, followed by the material cost and the fill pressure defects. Therefore, significant cost is generated because the product is not stiff enough and is likely to bend or break under the specified load. Applying the relations between the responses and the input parameter, it was determined that the thickness is the main driving input variable for those responses. By increasing the thickness to 1.95 mm, it was possible to reduce the defect cost due to the stiffness and fill pressure to $0.00, with only a slight increase in material cost to $0.25 and process cost to $0.04, minimizing the total cost of the product to $0.29 and creating a robust designed part.

Conclusion

The analysis of the cost drivers enables the development team to improve the design by focusing on the most significant contributions toward the cost. This analysis is measured in monetary units, making the results easy to understand and to estimate the impact of the different cost drivers. Future research includes an improved estimation and visualization of the significance of the input parameters towards the cost drivers, enhancing the understanding of the relation between the input parameter and the total cost. It is also intended to implement the shown methodology in a commercially available CAD software package.

Nomenclature

μ	Mean
σ	Standard Deviation
ϕ_x	Distributed Input Parameter
ϕ_y	Distributed Response
C_D	Cost due to Defects
$C_{De\neg j}$	Cost due to Defects excluding j
C_{Dej}	Cost reduction due to Defects excluding j
C_{Doj}	Cost due to Defects of Response j only
C_i	Cost due to a Single Response
C_M	Material Cost
C_{MP}	Marginal Part Cost
C_P	Process Cost

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C_T	Total Cost
E_i	Percentage Effect of Response I
i,j	Counters
k	Sample Counter
l	Experiment Run Counter
LSL	Lower Specification Limit
m	Sample Size
n	Number of Constrained Responses y
o	Number of Input Parameters x
P_i	Probability of Success for one Response
P_{Joint}	Joint Probability of Success
USL	Upper Specification Limit
x_i	Input Parameter
y_j	Output Response

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Key Words

Part Cost Driver, Probabilistic Utility, Yield, Quality Defects