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Flexible Design Methodology

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ABSTRACT

A flexible design methodology is developed to minimize the effect of uncontrolled variation by modeling potential design and manufacturing corrections in the product development process. Using this flexibility methodology, the different defect modes and the likelihood of these defects occurring is evaluated. For every defect mode, all design change options are investigated including the cost and probability of selection. The expected cost of the initial design including design changes is determined, thus allowing development of improved, more flexible designs. The theoretical method is demonstrated using an example. The results indicate that small changes in design variables may reduce the likelihood and cost of future design changes, yet provide opportunity for downstream cost minimization.

KEYWORDS

Flexible Design, Design Change Cost, Prediction Uncertainty, Robust Design

INTRODUCTION

The goal of robust design methodologies is to reduce the sensitivity of the design to variation. The robustness is typically evaluated using models, simulations or experiments. However, there is a possibility that the physical embodiment of the design might not satisfy the specifications due to uncertainties in the development process and lack of knowledge. Although the robust design prediction considers noise, it usually does not account for uncertainties and inaccuracies in the predictions of the design performance. Due to this lack of consideration for prediction inaccuracies, the finely tuned robust design might violate specifications because the underlying predictions lack the necessary accuracy. It is possible to model the uncertainty variation into the robust design evaluation to reduce the sensitivity to noise and uncertainty, yet this could increase the cost of the product while generating no value for the customer. The described methodology aims to minimize the expected cost of the design including development uncertainties.

Figure 1 shows a predicted feasible design region within which a design is assumed to be feasible. From this predicted design region, a design is selected according to the objective function. The objective function is shown in the graph as the diagonal contour lines. However, the actual design window and the feasible design window might not coincide completely due to uncertainties in the development process. If the selected design lies outside of the actual design window, a design change is necessary, even if the model predicted this design to be optimal. This required design change will result in unforeseen development costs, and may also alter the performance and cost of the product.

Figure 2 gives an overview of the flexible design methodology. The method starts by selecting an initial investigated design, for which a prediction model for the noise distributions and the uncertainty distributions in relation to the design variables are estimated. The possible defect modes for this design are determined with their likelihood of occurring. Based on these defect modes, the possible design changes are investigated, and their ability to resolve the defect mode analyzed. The part cost and change cost are used in conjunction with the improvement of the design change to estimate the likelihood of a design change being selected from the list of possible design changes for a given defect mode.

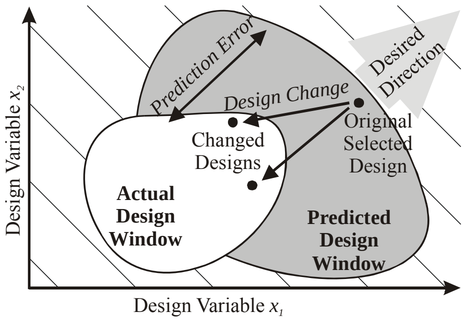

Figure 1: Flexible Design Incentive

Given all defect modes, the probability of a design variable being changed is determined and the expected cost of the design is evaluated. This information is then used to improve the initial design in order to enhance the flexibility of the design. This methodology will be demonstrated on an engineering example.

In order to evaluate the flexible design methodology, the prediction models of the design have to be known. The predicted relations between the design variables *X* and the design responses *Y* are required not only deterministically, but also probabilistically. Based on the noise distribution $pdf^N(X)$ of the design variables *X*, the noise distribution $pdf^N(Y)$ of the design responses *Y* has to be determined. A cost function for the marginal part cost is also required.

As the flexible design methodology does not only consider noise but also uncertainty, the prediction uncertainty distribution $pdf^U(Y)$ of the design responses *Y* have to be evaluated as a function of the error distributions *pdf(E)* and the deterministic responses *Y*. This prediction of the noise and uncertainty distributions can be performed using probabilistic methods. A functional evaluation (Papoulis 1991) is usually avoided due to the significant analytic effort and potential intractability. (Siddall 1986) describes probabilistic modeling in design by creating probability density functions. (Simpson et al. 1997) describes metamodels for probabilistic design predictions. Standard statistical distributions are frequently assumed to simplify the computation process using error transformations (Devore 1995). Monte Carlo methods are also commonly used (Suresh 1997). (Du and Chen 1999) provide an overview of different probabilistic evaluation techniques. The design system utilized in this paper is described in greater detail in (Roser 2000).

Other approaches for handling uncertainty are also described in the literature. (Thornton 1999) developed a method to analyze uncertainty regarding manufacturing process capability. (Dieck 1996) compares different measurement uncertainty models. (Thurston 1999) analyzes uncertainty in design performance attributes and its impact on decision-based design. (Otto and Antonsson 1993) compares the method of imprecision with utility theory for manipulating uncertainty. (Wood and Antonsson 1990) models imprecision and uncertainty in preliminary engineering design.

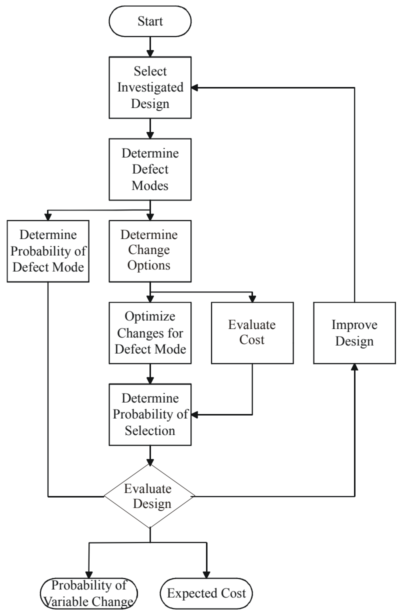

Figure 2: Methodology Outline

FLEXIBLE DESIGN METHODOLOGY

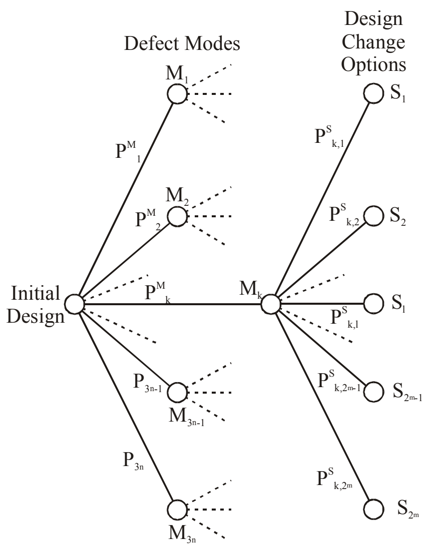

Figure 3: Defect Modes and Design Changes

For a selected initial design, there exists the possibility to have excessive defects due to prediction errors. Within this methodology, the possible types of defects are analyzed in detail. For any given defect, there exist different possible design changes in order to resolve the defect. An overview of the possible defect modes and design changes is given in Figure 3. The flexible design methodology investigates all possible design changes for all possible defect modes including the probability of occurrence to determine the flexibility of the design and to reduce the overall expected cost of the design including possible design changes.

Initial Design

The flexible design method analyzes a given initial design described by the initial design variables X^* and the resulting initial design responses Y^*. The selection of the initial design will determine subsequently the probability of the different defect modes and change options.

A design is feasible if all design responses Y are feasible. A design response y_j is feasible, if the probability of specification satisfaction is above a required limit, represented by a certain minimum distance α between the design response y_j and the lower and upper specification limits LSL^N_j and USL^N_j. This distance α is measured in standard deviations σ^N_j of the response noise distribution $pdf^N(y_j)$. A value of α of three would represent the probability of at least 99.7% of the responses y_j being within the specification limits LSL^N_j and USL^N_j despite noise variation. Figure 4 visualizes a feasible design response y_j with three deviations distance to both specification limits LSL^N_j and USL^N_j.

$$LSL_j^N + \alpha \cdot \sigma_j^N \leq y_j + e_j \leq USL_j^N - \alpha \cdot \sigma_j^N \quad \forall j$$

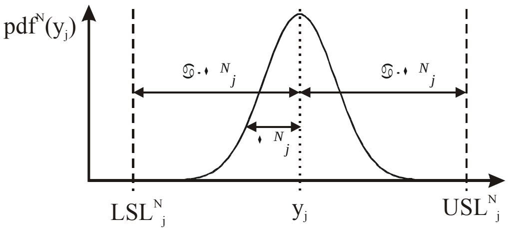

Figure 4: Quality Requirement

However, prediction uncertainties cause the actual response to differ from the predicted response y_j due to a prediction error e_j. There are numerous sources of uncertainty. (Chipman 1998) describes uncertainty in fitting models to experimental data using Bayesian methods. (Goodwin et al. 1990) also estimates model uncertainty using Bayesian networks. This uncertainty can be determined by comparing the prediction with either experimental data or high quality predictions throughout the investigated design space. (Steele et al. 1993) for example estimates precision uncertainty based on previous experiments. Unfortunately, the errors E for the prediction of the responses Y are not known beforehand, as otherwise, it would be possible to incorporate the errors E in the prediction of the responses Y to obtain accurate predictions. However, using statistical methods the probability distribution $pdf(E)$ of the errors E can be estimated.

The design team tries to ensure that the prediction errors E do not cause the mean responses Y to violate the quality criteria. Hence, the prediction including error has to be within the limits described by the specification limits LSL^N_j and USL^N_j, the standard deviation of the noise σ^N_j of the initial design, and the value of α. These new limits for the prediction error are nominated as the lower and upper specification limits for the uncertainty LSL^U_j and USL^U_j.

$$LSL_j^U = LSL_j^N + \alpha \cdot \sigma_j^N \quad \forall j$$
$$USL_j^U = USL_j^N - \alpha \cdot \sigma_j^N \quad \forall j$$

An initial design under uncertainty is assumed to be feasible if all response values *Y* including the prediction errors *E* are within the lower and upper specification limits for the uncertainty LSL^U_j and USL^U_j. The next section analyzes the possible defect modes of violating this quality requirement.

Defect Modes

Within this paper, a design is considered infeasible if one or more response y_j is outside of the specification limit LSL^U_j and USL^U_j of the uncertainty. This infeasibility may subsequently require a design change. This change depends on the exact nature of the defect. Therefore it is necessary to list all possible defect modes of the design and the probability of this defect occurring.

A design response y_j related to a two-sided specification limit LSL^N_j and USL^N_j could have three different defect modes under uncertainty. It is possible, that the response y_j violates the lower specification limit for uncertainty LSL^U_j. It is also possible that the response y_j violates the upper specification limit for uncertainty USL^U_j. Finally, it is possible that the response y_j violates none of the specification limits LSL^U_j and USL^U_j. This generates 3^n different possible defect modes for *n* responses.

These defect modes are summarized in a matrix *M*. This matrix consists of one column for each of the *n* specified response y_j and one row for each of the 3^n possible defect modes. A single defect mode is represented by one row M_k of the matrix *M*. The matrix element $M_{k,j}$ contains a –1 if the defect mode M_k violates the upper limit USL^U_j for a response y_j, and a +1 if the defect mode M_k violates the lower limit LSL^U_j for a response y_j. If the defect mode M_k does not violate any response under uncertainty, the matrix element $M_{k,j}$ contains 0. For n=2 *M* is shown below.

For two specified design responses y_j there are nine resulting defect modes M_k. The first defect mode M_1 represents a design, where the lower and upper specification limits LSL^U_j and USL^U_j under uncertainty for both responses y_j are satisfied. The second defect mode M_2 represents a violation of the upper specification limit for the second response y_2, while the first response y_1 is within the given specification limits. The last defect mode M_9 represents both responses y_j violating the lower specification limits LSL^U_j under uncertainty.

$$M = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ -1 & -1 \\ -1 & 1 \\ 1 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$$

The probability of the occurrence of a defect mode M_k depends on the probability of violating the individual specifications under uncertainty. A specification is violated if the response y_j including the prediction error e_j is outside the lower and upper specification limits for the uncertainty LSL^U_j and USL^U_j. As the prediction error is known only as a probability distribution $pdf(e_j)$, only the probability of specification violation can be estimated using the response probability distribution for uncertainty $pdf^U(y_j)$. The probability P^L_j of a response y_j violating the lower specification limit LSL^U_j or the probability P^U_j of violating the upper specification limit USL^U_j is shown in Figure 5. Note that for one-sided specifications the probability of violating the other side is zero, as a nonexistent specification cannot be violated. This can also be represented mathematically by setting the corresponding specifications to ±∞.

$$P_j^L = \int_{-\infty}^{LSL_j^U} pdf^U(y_j)dj \quad \forall j$$

$$P_j^U = \int_{USL_j^U}^{\infty} pdf^U(y_j)dj \quad \forall j$$

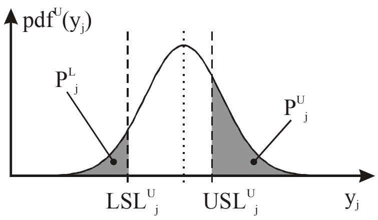

Figure 5: Defect Probability

Using the probabilities P^L_j and P^U_j, it is now possible to evaluate the probability P^M_k of a certain defect mode occurring. This probability is the combination of the individual probabilities of specification satisfaction or violation for each defect mode.

$$P_k^M = \left(\prod_{j=1}^{n} \begin{cases} P_j^U & if\ M_{k,j} = -1 \\ P_j^L & if\ M_{k,j} = 1 \\ 1 - \left[P_j^U + P_j^L\right] & else \end{cases} \right) + cov(\ ) \ \forall k$$

As the matrix M contains all possible defect modes M_k, the sum of the probabilities P^M_k of all defect modes M_k occurring has to be equal to one, i.e. the defect modes are mutually exclusive and completely exhaustive (Heckermann et al. 1995)

(Chipman 1998). This requirement is satisfied by the described probabilistic analysis. Figure 6 visualizes the Bayesian network of the different defect modes for a selected initial design. The next section will describe the possible design changes to resolve the defects for the different defect modes M_k.

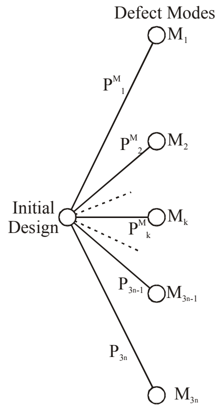

Figure 6: Defect Modes

Design Change

This section will now investigate different design change options for every defect mode M_k aimed to resolve the defect. For every possible design change S_l for every defect mode M_k the possible design improvement is determined and the probability of resolving the defect $P^R_{k,l}$ is evaluated. Subsequently, the probability $P^C_{k,l}$ of a change being utilized is extracted with respect to the total cost $C^T_{k,l}$.

If there exist m investigated design variables, there will be 2^m possible design change combinations S_l, representing 2^m subsets of the m dimensional design space. An example set S of possible design changes S_l for three design variables is shown below.

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Figure 7 represents the design space and subspaces for the above set of design changes S. The first change option S_1 represents changing no variables of X and keeping the initial design. This is represented by the zero dimensional sub space in Figure 7. The second design change option, S_2 represents only changing variable x_3, while keeping the variables x_1 and x_2 at the initial value. This is represented as the vertical one-dimensional sub-space in Figure 7.

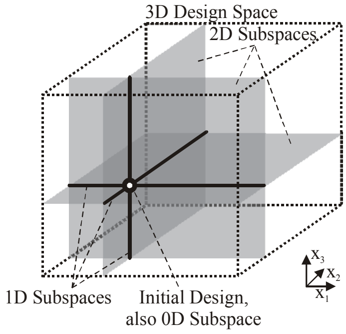

Figure 7: Design Space and Sub Spaces

Figure 8 shows an overview of the possible design change options. The design change options are identical for all defect modes. However, the actual design changes will differ from each other depending on the defect mode. The criteria used for the design change evaluation have to be described before the actual design change takes place.

Out of the 2^m design changes for the entire 3^n defect modes, some cases can be determined beforehand using common sense, subsequently reducing computation time. Some design responses y_i may have only a one sided specification. Therefore, the probability of violating the other specification is zero, and all defect modes M_k including this zero probability violation $P^L{}_k$ or $P^U{}_k$ have an occurrence probability $P^M{}_k$ of zero. Subsequently the probability of resolving the defect $P^R{}_{k,l}$ is set to zero for all design changes S_l for defect modes M_k. This is true for all defect modes M_k with zero probability of occurrence $P^M{}_k$.

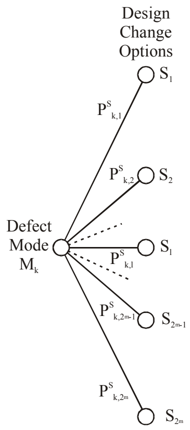

Figure 8: Design Change Options

$$P^R_{k,l} = 0 \ if \ P^M_k = 0 \ \forall l$$

Another defect mode M_k for which the design change is known beforehand is the first defect mode M_1, representing the case where all design variables Y are within the given uncertainty limits LSL^U and USL^U. If there is no violation of any specification limits under uncertainty, then there exists no need to change the design. Hence, the probability $P^R{}_{1,1}$ of resolving the (non-existent) defect by not changing the design is 1, and the probability of resolving the defect $P^R{}_{1,l}$ by means of any other design change option $S_{1,l}$ is zero.

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$$P_{1,1}^{R} = 1$$
$$P_{1,l}^{R} = 0 \ \forall (l > 1)$$

Finally, if a design is defective, it has to be changed. The option not to change the design will not resolve the problem. Hence, for any defect mode M_k with the exception of the first defect mode M_1 the option not to change the design $S_{k,1}$ has a probability $P^R_{k,1}$ of resolving the defect of zero.

$$P_{k,1}^{R} = 0 \ \ \forall (k > 1)$$

Otherwise, a given defect mode describes which design responses violate which side of the specifications. The goal of a possible design change S_l is to move the responses y_j violating the specification limits LSL^U and USL^U in a direction away from the violated specification limit, while keeping the non-violated responses within the specification limits LSL^U and USL^U. The change in the response value y_j for a given design change S_l and a given failure mode M_k is described as $\Delta y_{k,l,j}$. This change is measured in the direction away from the violated specification. Under the assumption that the prediction error remains constant, this will move the actual response away from the violated specification limit.

$$\Delta y_{k,l,j} = \left(y_j - y_j^*\right) \cdot M_{k,j} \quad \forall M_{k,j} \neq 0$$

An exact design change cannot yet be determined during the design development stage since the exact prediction error is not known. Instead, the probability of resolving all defects $P^R_{k,l}$ by means of changing the design is analyzed. This probability of resolving all defects $P^R_{k,l}$ is based on the probability $p^R_{k,l,j}$ of resolving the defect for one response y_j. A design change aimed to move the response away from the violated limit would reduce the probability of specification violation. This is visualized in Figure 9, where a response y_j is changed to reduce the probability of violating the lower specification limit under uncertainty.

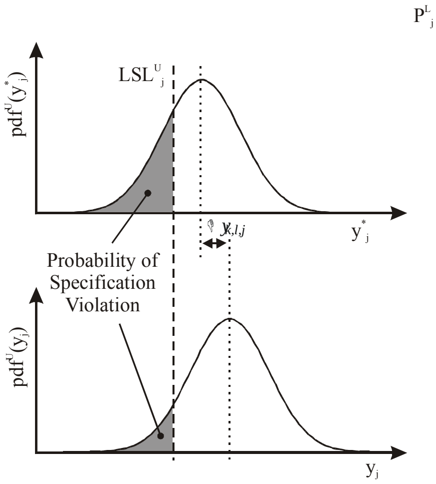

Figure 9: Probability of Specification Violation

The probability of resolving a specification violation under uncertainty $p^R_{k,l,j}$ by changing the design can be represented as the difference in the probability of violating the specification as a ratio of the initial probability of specification violation with a minimum probability of zero.

$$p^R_{k,l,j} = Max\left[0, \quad \begin{array}{ll} 1-\dfrac{\int_{USL_j^U}^{\infty} pdf^U\left(y_j\right)dy_j}{\int_{USL_j^U}^{\infty} pdf^U\left(y_j^*\right)dy_j^*} & if\ M_{k,j} = -1 \\ 1-\dfrac{\int_{\infty}^{LSL_j^U} pdf^U\left(y_j\right)dy_j}{\int_{\infty}^{LSL_j^U} pdf^U\left(y_j^*\right)dy_j^*} & if\ M_k, j = +1 \\ 1 & else \end{array}\right]$$

Based on the above probability of resolving one specification violation $p^R_{k,l,j}$ the joint probability of resolving all specification violations $P^R_{k,l}$ for a given defect mode M_k and design change S_l can be calculated.

$$P^R_{k,l} = \prod_{j=1}^{n} p^R_{k,l,j} + cov(\ )$$

Design Change Optimization

The design change aims to improve the probability of resolving all defects $P^R_{k,l}$ for a given defect mode M_k by means of a given design change S_l. All design responses, which do not violate the specification limit under uncertainty, have to remain within the uncertainty limits to ensure an acceptable quality. In addition, depending on the design change S_l only certain design variables x_i are allowed to change, whereas the other design variables x_i have to remain at the value of the initial design. This optimization has to be performed for every design change option S_l for every defect mode M_k excluding the known cases described above. Therefore, up to 3^n times 2^m optimizations have to be performed, and the probability of resolving the defect evaluated. For a review of different optimization methodologies please refer to (Reklaitis et al. 1983).

$$\begin{array}{ll} Max & P^R_{k,l} \\ & LSL_j^U \le y_j \le USL_j^U \quad if\ M_{k,j} = 0 \\ s.t. & LCL_i \le x_i \le UCL_i \quad if\ S_{l,i} = 1 \\ & x_i = x_i^* \quad if\ S_{l,i} = 0 \\ & k \in \left[1, 3^n\right], l \in \left[1, 2^m\right] \end{array}$$

Cost of the Design

In order to select between different design changes S_l capable of resolving a given defect M_k, the cost of the part including the cost of the design change to decide between different design change options is used.

The cost of changing the design variables has to be determined for all possible design parameter combinations S_l. This analysis requires the structuring of the tasks necessary to change a variable. This structure is related to design task modeling, where a development process is divided into sub tasks. (Steward 1981) describes the design structure matrix as an approach to manage complex design systems. This approach is extended for the change cost analysis.

The relation between the design variables x_i and the tasks ξ_l required for changing the design variables are represented in a matrix ξ. This matrix consists of one row for each design variable x_i, and one column for each possible task ξ_q. If a change in a design variable x_i requires the execution of task ξ_q, a one will be inserted in the matrix ξ in row i and column q. A matrix ξ representing the relation between three design variables and five tasks is shown below.

$$\xi = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

A task is required if at least one changed design variable requires the performance of the task. Furthermore, each task ξ_q creates a cost $C^{\xi}_{k,l}$ during execution. The change cost $C^{C}_{k,l}$ (per part) of the design change evaluates as the sum of the cost of all executed tasks divided by the production volume V.

$$C^{C}_{k,l} = \frac{\sum_{q=1}^{r} \begin{cases} C_r^{\xi} & if\ [S_l \cdot \xi]_q > 0 \\ 0 & else \end{cases}}{V}$$

Please note that the above method for determining the cost of a design change is a very general approach, and estimates the change cost merely based on the changed variables. The change cost might differ depending on the change direction or magnitude, for example if a hole diameter in a tool has to be reduced rather than increased. These asymmetric cost relations are not modeled within this system. Improved methodologies for change cost estimations can be developed and used within this methodology. Together with the marginal part cost $C^{M}_{k,l}$ the total cost $C^{T}_{k,l}$ of the design change S_l for a given failure mode M_k can be evaluated.

$$C^{T}_{k,l} = C^{M}_{k,l} + C^{C}_{k,l}$$

Probability of Design Change

After performing the above optimization for all possible design changes S_l and all possible defect modes M_k, a total of 3^n times 2^m design changes are evaluated. The 2^m design changes for a given defect mode M_k are summarized in Table 1.

Table 1: Design Changes for Defect Mode M_k

Design Change	Total Cost	Probability of Resolving

S_1	$C^T_{k,1}$	$P^R_{k,1}$
S_2	$C^T_{k,2}$	$P^R_{k,2}$
…		
S_l	$C^T_{k,l}$	$P^R_{k,l}$
S_{2m}	$C^T_{k,2m}$	$P^R_{k,2m}$

The above evaluation of the probability of resolving a defect $P^R_{k,l}$ and the total cost of a changed design $C^T_{k,l}$ can now be used to determine the likelihood of selecting a given design change from the set of possible design changes S for a given defect mode M_k. With respect to economic considerations, the different design changes S_l for a given defect mode M_k are sorted by the total cost $C^T_{k,l}$. The sorted list of possible design changes are indexed as t ranging from 1 to 2^m to differentiate it from the previous unsorted list indexed as l and also ranging from 1 to 2^m as shown in Table 2.

Table 2: Sorted Design Changes for Defect Mode $\mathrm{M_k}$

Design Change	**Total Cost**	**Sorting Criteria**	**Probability of Resolving**
$S_{k,1}$	$C^T_{k,1}$	-	$P^R_{k,1}$
$S_{k,2}$	$C^T_{k,2}$	$C^T_{k,2}>C^T_{k,1}$	$P^R_{k,2}$
…			
$S_{k,t}$	$C^T_{k,t}$	$C^T_{k,t}>C^T_{k,t-1}$	$P^R_{k,t}$
$S_{k,2}m$	$C^T_{k,2}m$	$C^T_{k,2}m>C^T_{k,2}m_{-1}$	$P^R_{k,2}m$

For a given prediction error E, there might be more than one design change $S_{k,t}$ capable of resolving the defect. With respect to economic considerations, the design change with the least total cost $C^T_{k,t}$ capable of resolving the defect mode M_k will be selected from the list of possible design changes S. Therefore, a design change would only be selected if the change $S_{k,t}$ resolves the defect and all more economic design changes do not resolve the defect.

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$$P_{k,t}^{S} = P\left(P_{k,t}^{R} \cap \overline{P_{k,t-1}^{R}} \cap \overline{P_{k,t-2}^{R}} \cap \cdots \cap \overline{P_{k,2}^{R}} \cap \overline{P_{k,1}^{R}} \right)$$

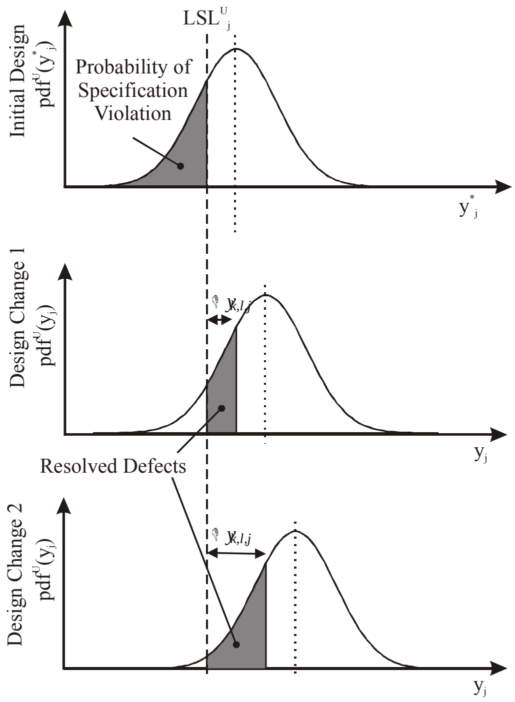

Figure 10: Interaction Assumption

To calculate the actual value of selecting a design change $S_{k,t}$ from the list of design changes S for a given defect mode M_k, interactions have to be taken under consideration and independence cannot be assumed. As shown in Figure 10 for a single violated response, design change two with a higher probability of resolving the defect $p^R_{k,2,j}$ will resolve all defects which where also resolved by design change one with a lower probability of resolving the defect $p^R_{k,1,j}$. Hence, the probability of change two resolving defects not resolved by change 1 is the difference between the two probabilities $p^R_{k,l,j}$ of resolving the defect. Vice versa, design 1 does not resolve any defects not resolved by design change two.

This assumption can be extended for the probability of resolving all defects $P^R_{k,t}$ to estimate the probability of selecting a given design change $P^S_{k,t}$ from the list of possible design changes S for a given defect mode M_k.

$$P_{k,t}^{S} = \begin{cases} P_{k,1}^{R} & if\ t = 1 \\ P_{k,t}^{R} - max\left(P_{k,t-1}^{R} \cdots P_{k,1}^{R}\right) & else \end{cases} \quad \forall k,t$$

The probability of a certain design change occurring $P^C_{k,t}$ depends on the probability of selecting this design change $P^S_{k,t}$ from the list of possible design changes S for a given defect mode M_k and the probability of the defect mode occurring P^M_k.

$$P_{k,t}^{C} = P_{k}^{M} \cdot P_{k,t}^{S} \qquad \forall k,t$$

Probability of Design Failure

However, there is the possibility of encountering a prediction error e_j, which is larger than the design can be adjusted for. This probability of not being able to satisfy the requirements is nominated as the probability of failure P^F for all defect modes M and can be determined as shown below.

$$P^{F} = 1 - \sum_{k=1}^{3^n} \sum_{t=1}^{2^m} P_{k,t}^{C}$$

The probability of design failure represents the likelihood of the initial design not being able to satisfy the quality requirements including the possibility of a design change, based on the available knowledge during the design development stage. The information regarding the probability of failure is restricted to the modeled design space. It may be possible to adjust parameters not modeled for the above methodology, or to extend the range of the design variables for example to include high quality material and processes not considered before. In addition, a change in the design concept might be able to resolve the defect despite large prediction errors. Finally, a relaxation of the quality requirements may resolve the defects. Therefore, a design failure as described in the above context does not necessarily mean the inability to create a design, which satisfies the customer requirements. Rather, it represents the inability to satisfy the quality requirements using the given design system and variation information.

In order to evaluate the expected cost of the initial design C^E, the probability of failure P^F has to be considered to model the defect modes M and design changes S. As the expected cost C^E is measured monetary, a cost has to be related to the design failure C^F. This cost is estimated from human expertise and may differ significantly depending on the circumstances of the design project.

Expected Cost

The expected cost C^E is the average cost of the design including all possible changes and failures for all defect modes. It is important, however, to note that this cost is a probabilistic average of different defect modes and design changes. As only one design is created, there will be only one certain defect mode with one design change. This case is not known until the design is created and the actual prediction errors E are known. Therefore, the design might cost more or less than expected, yet the average cost will be the expected cost C^E. The expected cost C^E for the initial design is the sum of all design costs $C^T_{k,t}$ including the probability of the change $P^C_{k,t}$, the probability of failure P^F and the cost of failure C^F.

$$C^E = \left(\sum_{k=1}^{3^n} \sum_{t=1}^{2^m} C^T_{k,t} \cdot P^C_{k,t} \right) + C^F \cdot P^F$$

Probability of Design Variable Change

Using the above probabilities, it is also possible to determine the likelihood P^X_i of changing a certain design variable x_i. The probability of a certain design change $P^C_{k,t}$ is known for all defect modes M_k and design changes S_l. Hence, it is possible to evaluate the probability of a variable change P^X_i by summarizing the probabilities $P^C_{k,t}$ of design changes $S_{k,t}$ including this variable x_i.

$$P_i^X = \sum_{k=1}^{3^n} \sum_{t=1}^{2^m} P^C_{k,t} \cdot S_{k,t,i}$$

Probability of Design Response Defect

The probability of a defect P^Y_j in a given response y_j can also be determined. This probability P^Y_j is the sum of the probabilities of violating the lower P^L_j and upper specification limit P^U_j under uncertainty.

$$P_j^Y = P_j^L + P_j^U$$

EXAMPLE

The demonstrated example is a large injection-molded part with a production requirement of 500,000 units. Four significant design variables from geometry, material, and processing parameters may be modified as necessary for the design of this product to deliver adequate response of four constrained quality attributes. These variables are shown in Table 3. Additional control variables are also specified within the prediction models, such as melt temperature, injection time, and different gating scenarios (single gate, multi gate and reverse ejection gate). Although these additional variables had significant impact on the design performance, they did not affect the optimal design using a cost or yield objective, i.e. the optimal design always used the same gating scenario, and other gating scenarios did not yield an optimal design. Hence, these variables were set to the desired value and excluded from the flexibility analysis to reduce computation time.

Table 3: Design Variables and Design Responses

Design Variable	**X**	**Design Response**	**Y**	**Constraints**
Mold Temperature	x_1	Melt Pressure	y_1	*USL*
Wall Thickness	x_2	Shrinkage	y_2	*LSL*, *USL*
Number of Tools	x_3	Clamp Force	y_3	*USL*
Material Type	x_4	Production Time	y_4	*USL*

The relations between the design variables and the design responses were modeled as response surfaces and functional evaluations using analysis, simulations and experimental data. A probabilistic evaluation was performed assuming normal distributions with error transformations.

The design variable with the most significant impact on the design is wall thickness. Thickness has a strong influence on all investigated design responses and the part cost. A reduced cost is achieved by minimizing the wall thickness. However, a reduced wall thickness increases the process cost and also increases the manufacturing difficulties. A design optimized for the cost objective is vulnerable to prediction uncertainties, since it is likely that the wall thickness has to be increased in case of an infeasible design. This requires costly and timely retooling. To avoid this problem the wall thickness can be increased to improve the robustness. However, this robustness also increases the part cost. The flexible design methodology analyzes the trade off between part cost and change cost. The initial design for the methodology is the design optimized for the cost subject to the quality requirements. The selected initial design has a marginal part cost of $5.91.

As there are four design responses, there will be 81 defect modes. However, as three responses are limited on only one side, this can be reduced to 24 possible defect modes. The seven most common defect modes are listed in Table 4, sorted by the probability of appearance P^M_k. All remaining defect modes occur with less than .1% likelihood.

Table 4: Significant Defect Modes

Index	y_1	y_2	y_3	y_4	P^M_k
3	0	0	-1	0	31.7
1	0	0	0	0	21.8
4	0	0	-1	-1	19.5
2	0	0	0	-1	13.4
15	-1	0	-1	0	5.0
16	-1	0	-1	-1	3.1
14	-1	0	0	-1	2.1

For each defect mode, there are 16 possible design changes, ranging from no change to changing all of the four design variables. All design changes for all defect modes where analyzed as described above. Table 5 shows the likelihood of a certain response violating the quality requirements due to uncertainty. There is a large possibility of violating the clamp force and the production time for the given initial design.

Table 5:Probability of Response Defect

Design Response	P^X_i
Melt Pressure	13.6
Shrinkage	0.0
Clamp Force	59.3
Production Time	38.1

The resulting probabilities of variable changes are shown in Table 6. The most frequently changed variable is the mold temperature. However, the wall thickness and the number of tools is also changed frequently, both are comparatively expensive design changes. These expensive design changes increase the expected cost of the design.

Table 6: Probability of Variable Change

Design Variable	P^X_i
Mold Temperature	56.9
Wall Thickness	13.9
Number of Tools	22.3
Material Type	0.1

Table 7 shows the design evaluation. It can be seen, that there is only a small change of 21% of the initial design being acceptable as is due to the possible prediction errors. There is a change of over 60% that the design has to be changed, and an additional 15% change of the design being infeasible. The expected cost is $7.68.

Table 7: Design Evaluation

Probability of no Change $P^C_{1,1}$	21.8 %
Probability of Any Change	62.5 %
Probability of Failure	15.7 %
Expected Cost	$ 7.68

The initial design was changed to reduce the sensitivity to uncertainty. The wall thickness was increased to reduce the likelihood of violating the clamp force and melt pressure requirements. The mold temperature was reduced to reduce the likelihood of violating the production time requirements. Table 8 shows the probability of a defect in a response for the improved model. Comparing these values to the first design shown in Table 5 shows no large differences in the probabilities. This design has only a slightly reduced probability of occurring defects except for the production time.

Table 8: Probability of Response Defect (Improved Design)

Design Response	P^X_i
Melt Pressure	13.1
Shrinkage	0.0
Clamp Force	55.8
Production Time	15.6

However, an important difference can be seen in the way to resolve these defects. Table 9 shows that the probability of changing the wall thickness or increasing the number of tools is significantly reduced as compared to Table 6. Although there is still a significant number of design changes expected, almost all of these defects can be resolved by means of a fast and economic change in the mold temperature. This is represented in Table 10, where the expected cost at $6.78 is $0.90 lower the $7.68 of the previous design, saving an estimated $450,000 for the number of produced parts.

Table 9: Probability of Variable Change (Improved Design)

Design Variable	P^X_i
Mold Temperature	59.4
Wall Thickness	3.9
Number of Tools	8.1
Material Type	0.0

Table 10: Design Evaluation (Improved Design)

Probability of no Change $P^C_{1,1}$	32.4 %
Probability of Any Change	60.1 %
Probability of Failure	7.5 %
Expected Cost	$ 6.78

SUMMARY

The flexible design methodology aims to reduce the overall cost by reducing the cost of design changes due to prediction errors. The goal is not to improve the robustness against uncertainty, but rather to reduce the negative impact of uncertainty on the cost of the design. The method evaluates the possible defect modes due to uncertainty and analyzes the possible design changes. Thus, it predicts for the design team the likelihood of certain defects, and which variables would have to be changed in order to resolve the defects. This enables the design team to modify the design in order to improve the flexibility of the design, resolving defects using economic design changes instead of costly and delaying design changes.

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