A FLEXIBLE DESIGN METHODOLOGY

A Dissertation Presented

by

CHRISTOPH H. ROSER

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DEDICATION

To My Parents

EPIGRAPH

"*Daß ich erkenne, was die Welt*
Im Innersten zusammenhält…"

Faust

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ABSTRACT

A FLEXIBLE DESIGN METHODOLOGY

MAY 2000

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The flexible design methodology facilitates development of a minimal cost design that can later be adjusted for possible prediction uncertainties. Uncertainty arises in product development from assumptions and simplifications in the performance model, lack of certainty regarding the requirements, or simply errors in design development. Depending on the scope and impact of uncertainty, the design might have to be changed later in the product development cycle with negative impact on cost, performance, and development time. For a given design, the flexible design methodology determines all possible expected outcomes and their likelihood of occurrence. For each expected outcome, the likelihood and cost of all possible design changes are evaluated. An expected cost is then derived from analysis of the change costs and their joint probabilities of the expected outcomes and design changes. The expected cost, together with the failure probability, can assist the designer in developing a flexible design. The described methodology not only facilitates the trade-off between minimal cost and risk, but also evaluates the potential benefit of prediction models with improved prediction accuracies. The results indicate that a flexible design allows quick adjustment for uncertainties, with only a slight increase in the marginal part cost.

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μ	Mean of a Probability Distribution
σ	Deviation of a Probability Distribution
ξ	Design Change Task Matrix
α	Distance between Specification Limits for Noise and Specification Limits for Uncertainty
$\sigma^N{}_j$	Design Response Deviation under Noise
$\mu^N{}_j$	Design Response Mean under Noise
$C^C{}_{k,l}$	Change Cost of a Design for an Expected Outcome and a Design Change
$cdf^S(y_{k,l,j})$	Cumulative Density Function of Design Change Success
C^E	Expected Cost of the Design
C^{EI}	Expected Cost of Design including Cost of Information
C^I	Cost of Information
C^M	Marginal Part Cost
C^S	Cost of a Successful Design Change
C^U	Cost of an Unsuccessful Design Change
cov	Covariance of an Equation
$C^T{}_{k,l}$	Total Cost of a Design including Design Changes for an Expected Outcome and a Design Change Cost
E	Vector of Prediction Errors
f	Functional Relation between the Design Variables and the Design Responses
g	Functional Relation between the Design Variables and the Marginal Part Cost
h	Probabilistic Relation between the Design Variable Probability Density Distribution and the Design Response Probability Density Distribution

i	Index for Design Variables
j	Index for Design Responses
k	Index for Expected Outcomes
l	Index for Design Changes
LCL	Lower Constraint Limit
LSL	Lower Specification Limit under Noise
LSL^{N}_{j}	Lower Specification Limit under Uncertainty
M	Matrix of Expected Outcomes
m	Number of Design Variables
n	Number of Design Responses
P^{α}	Probability of Satisfying One Specification according to Quality Requirement
$P^{C}_{k,t}$	Probability of a Design Change Occurring
$pdf(E)$	Probability Density Function of the Prediction Error
$pdf^{N}(X)$	Probability Density Function of the Design Variable Noise
$pdf^{N}(Y)$	Probability Density Function of the Design Response Noise
$pdf^{S}(y_{k,l,j})$	Probability Density Function of Design Change Success
$pdf^{U}(Y)$	Probability Density Function of the Design Response Uncertainty
$P^{D}_{k,l}$	Probability of Satisfying All Specifications for an Expected Outcome and a Design Change
$P^{D}_{k,l,j}$	Probability of Satisfying One Specifications for an Expected Outcome and a Design Change
P^{F}	Overall Probability of Failure
P^{F}_{k}	Probability of Failure for a given Expected Outcome
P^{L}_{j}	Probability of Violating Lower Specification Limit under Uncertainty
P^{M}	Probability of an Expected Outcome Occurring

$P^M{}_k$	Probability of an Expected Outcome Occurring
P^N	Probability of Satisfying All Response Specifications under Noise
$P^N{}_j$	Probability of Satisfying a Response Specification under Noise
$P^S{}_{k,t'}$	Probability of a Design Change Occurring for a given Expected Outcome
$P^U{}_j$	Probability of Violating Upper Specification Limit under Uncertainty
$P^{\mathrm{X}}{}_{\mathrm{i}}$	Probability of Design Variable Change
$P^Y{}_j$	Probability of Violating a Specification
q	Index for Design Change Tasks
r	Number of Design Change Tasks
S	Matrix of Design Change Options
S/N_L	Signal to Noise Ratio
t	Index for Sorted Design Changes
u	Index for Prediction Models
UCL	Upper Constraint Limit
USL	Upper Specification Limit under Noise
$USL^N{}_j$	Upper Specification Limit under Uncertainty
V	Production Volume
X	Vector of Design Variables
$\mathrm{x_i}$	Design Variable
Y	Vector of Design Responses
Y^*	Actual Design Responses
y_j	Design Response

GLOSSARY

(Parker 1994) and (Webster 1988) were a valuable source of definitions, from which many of the following definitions were cited completely or with modifications. Prof. Wei Chen from the University of Illinois at Chicago, Prof. Linda Schmidt from the University of Maryland and Prof. Kemper Lewis from the State University of New York at Buffalo also provided valuable assistance through their decision based workshop.

Assumption:Information generated without sufficient proof of correctness.

Constraint: Limit on the design variable.

Cost, Change: Sum of all costs required to change a design.

Cost, Expected: Cost of a design including cost of possible changes.

Cost, Information: Cost of acquiring and verifying information.

Cost, Marginal: Cost of creating one instance of a design. Consists of the material cost, the process cost and the amortized tool cost.

Cost, Total: Sum of marginal cost and change cost.

Cost: Monetary measurement of value.

Expected Outcome: Unique combination of specification violations of all specified responses.

Design Change: Change to the design variables of a design.

Design Element: Generic superset of design variable, response, or specification. Includes for example wall thickness, process temperature, deflection and weight requirements.

Design Flexibility: Relative ability of the design response to be changed with a small change effort.

Design Instance: One individual part or unit created from a design.

Design Response: Metric in an engineering design dependent on or more design variables, which is specified to satisfy certain criteria. Common specifications require the design response to be more and/or less than an specification limit or to be equal to a target specification. A distinction is made between the actual response measured from a large, ideally infinite, number of samples and considered to be the true design response of the design and the predicted design response, estimated using a prediction model. The estimated design response may differ from the actual design response and may also differ for different prediction models.

Design Space: Multidimensional space created by the design variables. The boundaries of the design space are defined by the specification limits. The design variables can have the set of values of any point within the design space.

Design Variable: Controlled or uncontrolled variable influencing an effect in an engineering design. Can be described using a metric or subjective measurement. Within this dissertation only controlled design variables are discussed. Design variables may include but are not limited to processing variables, material properties, part and tool geometry.

Design: Creation that embodies ideas, aims and objectives. To create, fashion, execute, or construct according to a plan.

Error: Error is the use of incorrect information or the incorrect use of information.

Feasibility: A design satisfying all response specifications.

Information: The understanding of facts, data and observable relations. Information may be inaccurate, incomplete or wrong.

Knowledge: Within this dissertation used synonymously with information.

Method: A systematic procedure, technique, or mode of inquiry employed by or proper to a particular discipline or art.

Methodology: A body of methods, rules, and postulates employed by a discipline. A particular procedure or set of procedures.

Noise: Uncontrolled variation. Lack of control may be due to the general inability to control a variable or due to economic or other considerations against the controlling of a variable. Includes for example friction or vibration, which may not be controlled explicitly.

Objective: Description or measurement of one or more desired design properties for comparison of different designs.

Prediction Error: Difference between the predicted mean design responses and the actual mean design responses.

Prediction Model: A system of postulates, data, and inferences presented as a mathematical description of an entity in order to predict the design responses based on the design variables without creating one or more instances of the design.

Probability Density Function: A function of a continuous random variable whose integral over an interval gives the probability that its value will fall within the interval.

Quality Requirement: Required probability of specification satisfaction despite noise variation.

Risk: The potential realization of undesirable consequences from hazards arising from a possible event. In relation to engineering design, risk is the possibility of an undesired outcome of the design. The level of risk is related to the probability of the occurrence of an undesired outcome and the negative effects due to this undesired outcome.

Robustness: General insensitivity to variation, where the variation has either little effect on the design performance or little negative effect on the design performance.

Simplification: The ignorance of available information in order to reduce the effort of handling this information.

Specification Satisfaction: Situation, where the mean design response is within the range(s) defined by the specification limits.

Specification Violation: Situation, where the mean design response is not within the range(s) defined by the specification limits.

Specification: One or more metrics associated with a design response using Boolean logic. The outcome of the combinatory rule has to be true in order to satisfy the specification. Common combinatory rules may for example require the response to be larger or smaller than the metric, to be between or outside the range of two metrics or to be equal the metric. Includes for example upper weight limits, upper and lower deflection requirements. A specification is feasible if the design response is within the allowed range and infeasible otherwise.

Task: Action within a set of actions designed to achieve a desired goal.

Trade-Off: A balancing of factors all of which are not attainable at the same time.

Uncertainty: Lack of knowledge. In relation to engineering design, uncertainty is a lack of knowledge about the causes, effects, and their relations in a design. The level of uncertainty can range from a total lack of knowledge to the complete knowledge about a design element, in which case there would be no uncertainty at all.

Yield: Measurement for the probability of a part satisfying all specifications based on the design response variations and the specification limits. The yield can also be expressed in different ways for example by the number of defect parts per million parts.

CHAPTER 1

INTRODUCTION

1.1 Development of Engineering Design

The objective of this research is to develop a methodology for analyzing design responses under uncertainty to assist in the development of an economic design with the flexibility to adjust for prediction uncertainty. A brief review of the development of the fundamental engineering design methods is outlined in Figure 1. Each advancement leads to the improvement of the previously used methods, and to resolve shortcomings of the previous methods. Each of these fundamental methods is still utilized in varying degrees, and often in combination with each other. This list claims neither to be complete nor to be in a precise order. In addition, it can only be speculated what the next important developments in engineering design are. However, the author believes that the handling of uncertainty will be one of the next important tasks for engineering design research. The flexible design methodology attacks this design problem, which has received little attention yet, and the research presented in this dissertation will contribute to the field of engineering design research.

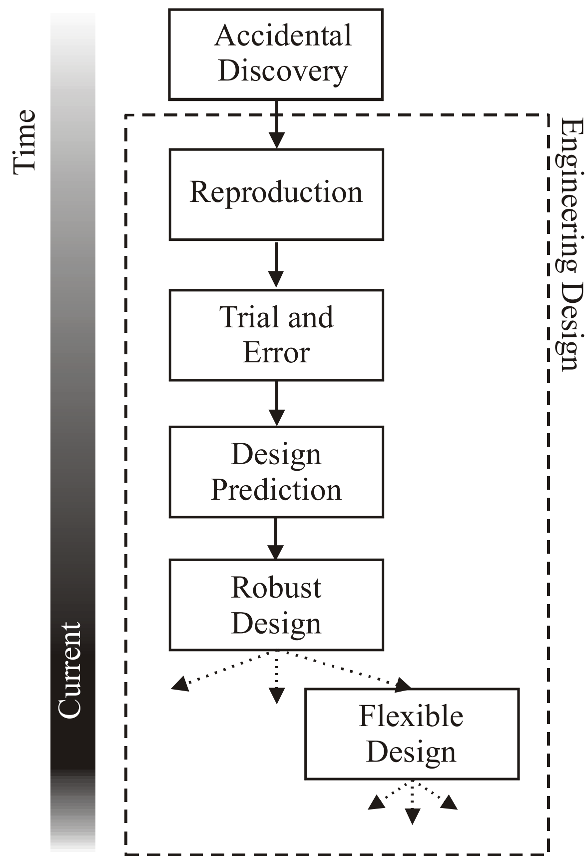


Figure 1: Development of Engineering Design

1.1.1 Reproduction

Engineering design utilizes a systematic approach to achieve a desired function in a product. Without a desired objective and a logical approach, it is not an engineering design process. When, about 2 million years ago, a homo erectus broke a stone by accident, and then figured out that he/she can cut things with the sharp edge, he/she didn't use engineering design but rather made an accidental invention.

However, when this early kind of man decided to make another "knife", basic engineering methodology was used. The homo erectus decided to reproduce a similar stone, with the objective to get a similar sharp edge usable for cutting. With this problem defined, he/she utilized the knowledge gained by the accidental production of the first knife, and tried to break another stone in the same way. Therefore, the engineering method of reproducing existing objects was utilized.

This method was used frequently throughout history, where for example feudal Japan reproduced Portuguese muskets, and is still in use today. The goal of reproduction, however, has shifted slightly from reproducing to understanding an existing design. This process is known as reverse engineering, and remains a subject of modern design research (Otto and Wood 1996).

1.1.2 Trial and Error

The objective of engineering design is not only to reproduce an existing design, but also to create a new design, existing only in the imagination of the designers. One of the simplest engineering methods is trial and error. In the case of the homo erectus, the idea might have been a stone knife with a comfortable grip, or a specially shaped edge for a special purpose. Due to the lack of other engineering methods, trial and error may have been used to obtain a desired stone, whose design then could have been reproduced.

Trial and error was one of the most commonly used methods of design development up to the industrial revolution. Goodyear, for example, spent years of his life and literally all of his fortune for trial and error experiments in order to develop the vulcanization of rubber in 1844 (Hubert 1893).

The method of trial and error is still in use today, frequently in combination with other methods, although it is usually highly ineffective and not very desirable. It is usually used if a process is complex and not very well understood.

1.1.3 Deterministic Design Prediction

The trial and error approach is not very effective due to the excessive cost of trying different designs until a feasible design is found. Instead, investing the effort to understand the underlying design relations facilitates determination of new designs according to rational decision-making.

Whereas the previous engineering methods do not involve science, the design prediction uses science to predict the behavior and engineering to create a design. An outstanding individual in this respect was Leonardo da Vinci, who not only understood the science, but also created new and unique engineering designs based on this science. For example, he adapted the Archimedes wheel to design a helicopter, and he designed the parachute, very much similar to as it is used today (Letze and Buchsteiner 1997).

Undoubtedly, the ability to predict the behavior of an engineering design by the use of prediction models is crucial to any modern engineering design. For example, any text for engineering statics includes prediction equations derived from scientific analysis.

1.1.4 Robust Design

However, the deterministic design process does not always create consistent product quality, as the production capabilities are limited, and causes the behavior of different instances of the design to vary slightly. To resolve this issue, the research community developed methods to handle noise variation in engineering design. These methods are frequently described as robust design (Chen and Choi 1996; Esterman et al. 1996; Iyer and Krishnamurty 1998; Roser and Kazmer 1998; Taguchi 1979; Taguchi and Clausing 1990; Taguchi and Phadke 1984; Thornton 1999c), and many different flavors of robust design have been developed and are still under development. Currently, 40 years after the introduction of the robust design methodology in the research community, the adaptation of robust design methods in industry is still in progress.

The second chapter describes a robust design method without consideration of the prediction uncertainty. This chapter also introduces a simple engineering example used to demonstrate the flexible design methodology. Chapter 3 analyzes the different sources of prediction uncertainty, and the effects of prediction uncertainty for the quality of the design and the possibilities of design change. However, this chapter does not yet provide an approach for handling uncertainty.

1.1.5 Flexible Design

In most engineering design methods, prediction models are used. Although it is usually known that these models may not be absolutely accurate, uncertainty is frequently disregarded due to lack of methods to handle these inaccuracies. Research regarding the handling of uncertainty is still in its infancy, and the utilization in industry is virtually nonexistent. However, the author believes the impact of the uncertainty on engineering design is significant and must be addressed. The problem is described below in this chapter in greater detail, and the solution approach is outlined.

1.2 Presented Methodology

Figure 2 shows the outline of a design system as used within this dissertation. The design variables, subject to noise, are evaluated using a prediction model, which predicts the design responses with certain accuracy. The predicted responses are subject to the specifications in order to evaluate the quality and the objective of the design.

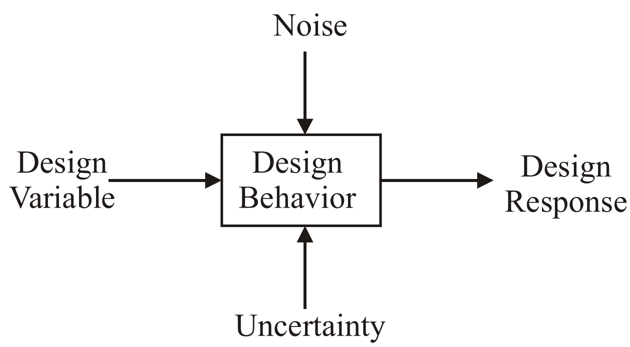


Figure 2: Design System Outline

1.2.1 Problem Statement

Engineering design utilizes design predictions to determine the behavior of a design without the effort of creating the physical embodiment of the design. However, as mentioned above, these design predictions may have inaccuracies, where the actual design response differs from the predicted design response. For very small errors, the effect on the design might be insignificant. However, for larger errors there are two possible effects, each of which is undesirable. First, the prediction error may cause an increase in the defect rate since the actual response violates or is closer to a specification limit than predicted. Second, if the prediction error moves the response away from the specification limit, the number of defects is reduced. In this case, however, it might have been possible to reduce the cost of the design by reducing this excessive robustness against uncertainty.

In either case, the prediction error reduces the knowledge of the design team about the behavior of the design. There are two current methods to avoid defects due to prediction uncertainty. First, the design may be created according to the prediction models with complete disregard of the prediction uncertainty. In case of an excessive number of defects due to uncertainty, the design might have to be changed, delaying the design and adding cost to the design project. Second, it may be possible to create a design that is insensitive to prediction uncertainty, where the expected prediction uncertainty will not cause additional defects. However, in this case the design is likely to be overly expensive in order to provide the reduced sensitivity to uncertainty. Therefore, there exists a dilemma between the cost of the robustness and the cost of the changes and defects.

The flexible design methodology described in this dissertation aims to generate a trade-off between the cost of design changes and the cost of the design. This trade-off should be determined in the design development stage in order to select and build the design with the least expected cost including possible design changes. The necessary steps are described in more detail in the following.

1.2.2 Uncertainty Description and Requirement

In order to estimate the number of defects due to prediction uncertainty, the prediction uncertainty has to be modeled and a quality requirement has to be defined. Within this dissertation, probabilistic methods are used to describe a probability density function of the prediction uncertainty and to evaluate the effects of the uncertainty. The second chapter describes a robust design method without consideration of the prediction uncertainty. A simple engineering example is introduced to demonstrate the flexible design methodology. Chapter 3 analyzes the different sources of prediction uncertainty, and the effects of prediction uncertainty upon the quality of the design.

1.2.3 Possible Defects

Using the uncertainty models, the probability of violating the specifications can be determined, i.e. the likelihood of the prediction uncertainty to require a design change. Within this dissertation, a distinction is made between the different expected outcomes, and the probability of a defect occurring is evaluated. Depending on the type of the defect, the feasible design changes may differ significantly. This flexible design methodology evaluates all possible expected outcomes exhaustively as described in Chapter 4.

1.2.4 Possible Design Changes

If the specifications are violated due to prediction uncertainty, the design has to be changed. There are different options for the design change. Depending on the number of design variables, any combination of design variables can be changed. Each combination of changed variables, however, has its individual cost of change, representing the effort to change the design variables. In addition, the effect of a design change depends on the investigated expected outcome and the changed variables.

As described in Chapter 4, the flexible design methodology considers all possible design changes for all possible expected outcomes a priori to determine the likelihood of the design change and the cost of the changed design. This exhaustive analysis of the defects and associated design changes allows the prediction of the overall expected cost. In turn, this measurement of the expected cost allows the comparison of different candidate designs in order to determine the design with the least expected cost, representing the best trade-off between the cost of design changes and the cost of reduced sensitivity to uncertainty. Two methods for the design change analysis are provided. In Chapter 4, a deterministic method is utilized, where the outcome of a design change is assumed to be known exactly. However, this is usually not the case in industry, where a design change might or might not resolve a defect. A design change analysis under uncertainty is described in Chapter 5. This improved design change analysis can be used if the information regarding the design change uncertainty is known, otherwise the deterministic approach may be utilized without significant loss of accuracy.

1.2.5 Design Flexibility

It is the expectation that the design with the least expected cost frequently may be very flexible, depending on the level of uncertainty and the cost and impact of the design changes. The effort required to change design variables depends on the system physics, manufacturing facilities, and engineering processes. While in general it is desirable to avoid the change of any design variables, this is especially true for expensive design variables. Yet, to reduce the probability of change for all design variables could require an excessive cost in the design.

However, if a design variable is relatively inexpensive to change, it may be beneficial to use this variable to compensate for prediction uncertainty. If the prediction error causes a violation of the specifications, this easy-to-change design variable may be used to adjust the prediction error in order to obtain a feasible design. Of course, this is only possible if this variable has the flexibility to be changed and also has an impact on the violated design responses. The flexible design methodology seeks to facilitate a design where ideally a few easy-to-change design variables are used to adjust for the prediction error, and the design variables with a large change cost remain unchanged, creating a design with an economic trade-off between the cost of the design and the cost and likelihood of design changes. In summary, a flexible design does not try to avoid design changes at all cost, but only aims to avoid expensive design changes. Inexpensive design changes are valuable degrees of freedom in a design, allowing a low cost design with the ability to adjust for prediction uncertainties fast and economically.

1.2.6 Value of Information

The flexible design methodology evaluates the expected cost of a design including possible design changes with respect to the prediction uncertainty. The methodology also enables the comparison of different models with different prediction accuracies in order to determine the value of the prediction accuracy, i.e. the value of information. If the possible benefits of a prediction model would be known, it is possible to compare these benefits with the cost of creating this prediction model.

As determined in Chapter 6, the estimated model accuracy is simulated using a readily available but less accurate prediction model. The expected mean and standard deviation of the prediction errors of the different prediction models are estimated. The flexible design methodology can then evaluate the models and assess the value of reduced uncertainty.

CHAPTER 2

ROBUST DESIGN FOR NOISE

2.1 Chapter Overview

The chapter first describes the deterministic design approach. The chapter then describes the underlying robust design approach used within this dissertation, as related to selected robust design methods. The goal of this chapter is to describe methods used throughout the remainder of the dissertation. As such, this chapter will make use of probabilistic methods, but leave the detailed description of these methods to other references. A selection of references is listed where appropriate.

A schematic of the robust design method used within this dissertation is shown in Figure 3. A probabilistic design evaluation is used to predict the distribution of the design responses as a function of the design variables subject to noise. The yield is evaluated as the joint probability of the design responses meeting the specification limits. The marginal part cost is then used as an objective to improve the design subject to a quality requirement. The chapter closes by introducing an example that will be used throughout the dissertation.

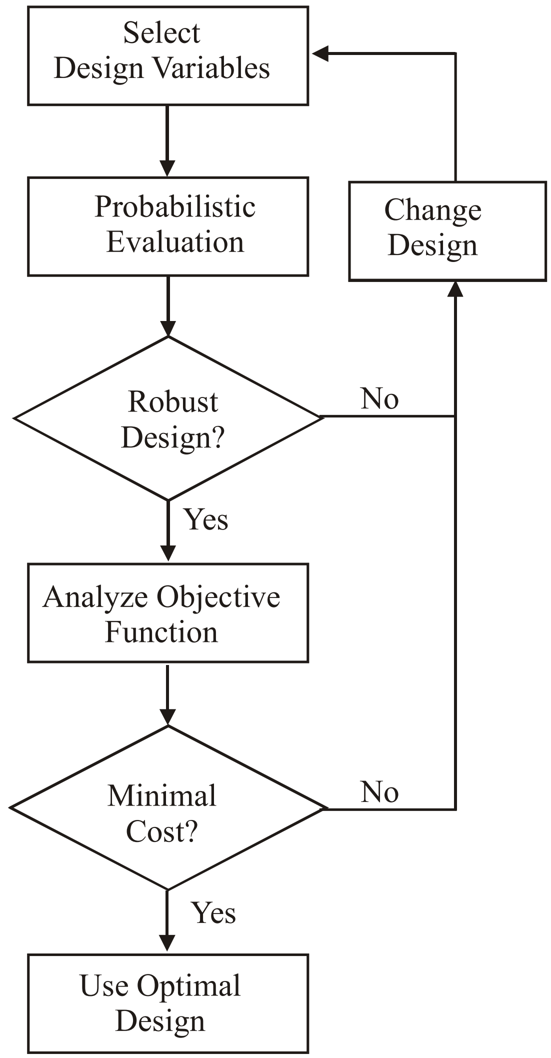


Figure 3: Robust Design Methodology for Noise

2.2 Design Approaches

This section introduces the deterministic design approach and the subsequent probabilistic design approach. As these sections do not describe new knowledge but rather only lay the foundation for the flexible design methodology, the description is brief and references are cited for more detail.

2.2.1 Deterministic Design Approach

The goal of engineering design is to create a design where all design responses *y* represented by the vector of design responses *Y* are within the lower and upper specification limits *LSL* and *USL*, where the design responses *Y* are a function of the design variables *X* as shown in Equation 1. Design variables are not restricted to geometry but may include processing variables and material properties.

$$Y = f(X)$$

Equation 1

Within this dissertation, prediction models representing these relations are required for the application of the developed methodologies. A more detailed description of the prediction models is omitted here, as this depends on the underlying design relations. The design team must determine the value of the design variables, *X*, that generates the design responses, *Y*, to be within the given specification limits as represented in Equation 2.

$$LSL \leq Y \leq USL$$

Equation 2

Unfortunately, the relations between the design variables, *X*, and the design responses, *Y*, are frequently complex. In addition, a change in the design variables *X* might move one design response *y* inside the related specification limits, but at the same time move another design response *y* outside of the related specification limits. In a deterministic design approach, the goal is to move all design responses in Y within the given specification limits, where the design variables X and the design responses Y are considered deterministic values. Frequently there exists more than one feasible design, allowing the design team to choose from different feasible designs. In this case, an additional objective can be used to represent the desirability of a given design, and subsequently to select the most desirable design from the feasible set of designs. Within this dissertation, a cost objective is used to describe the benefit of the design. For the deterministic case, this cost is represented by the marginal cost C^M measured per part as a function of the design variables X as shown in Equation 3.

$$C^M = g(X)$$

Equation 3

This marginal part cost may consist of material costs, processing costs and amortized tooling costs. Within this dissertation, a cost model representing the relation is required for the evaluation of the developed methodologies. A detailed description of the cost modeling is avoided, as the model form depends on the specific engineering application.

2.2.2 Probabilistic Design Approach

An engineering design is frequently used for mass production, where the number of parts produced may exceed millions. Unfortunately, it is extremely difficult to create absolute identical design instances twice, let alone millions of times. Rather, the values in the design variables will vary slightly, also causing the dependent design responses to vary. This variation may cause a feasible deterministic design to produce infeasible instances of the design, as the variation may move one or more design responses in *Y* outside of the related specification limits as visualized in Figure 4 for a design with one design variable *x* and one design response *y*. In this figure, the deterministic response is between the specification limits. However, due to noise in the design variable, the probabilistic response of the manufactured product may violate the lower and upper specification limits.

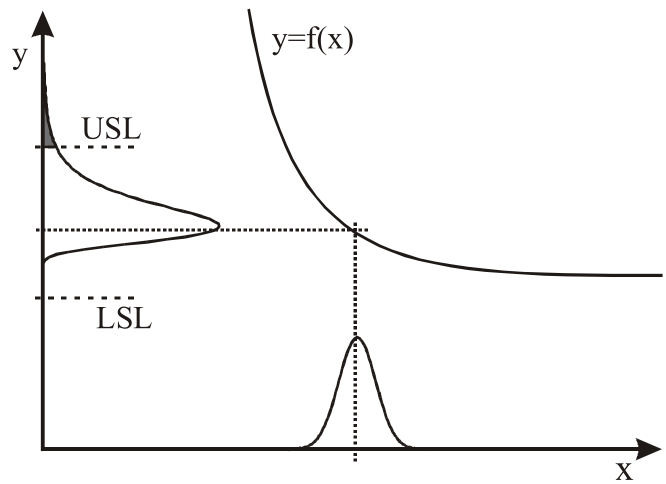


Figure 4: Design Variation

The method of robust design is used to reduce the effect of variation on critical design responses. There are two different fundamental approaches to robust design. (Parkinson 1995) describes them as feasibility robustness and sensitivity robustness. The original approach developed by Taguchi (Dehnad 1989; Taguchi 1993a; Taguchi 1993b; Taguchi and Konishi 1992; Taguchi and Phadke 1984; Taguchi and Wu 1985) uses a signal to noise ratio S/N_L to minimize the effect of design response variation as shown in Figure 5. (Chen et al. 1996) develops a similar method to minimize the variation caused by noise variables and control variables. Equation 4 shows the mathematical evaluation of the signal to noise ratio S/N_L for a set of response values y_i ranging from 1 to n_r (Schmidt and Launsby 1994).

$$S/N_L = 10Log_{10}\frac{1}{n_r}\sum_{i=1}^{n_r}\left(\frac{1}{y_j^2}\right) \quad \textit{for maximizing the response}$$

$$S/N_L = 10Log_{10}\frac{1}{n_r}\sum_{i=1}^{n_r}\left(y_j^2\right) \quad \textit{for minimizing the response}$$

$$S/N_L = 10Log_{10}\frac{1}{n_r}\left(\frac{S_m - V_e}{V_e}\right) \quad \textit{for a target response}$$

$$\textit{where } S_m = -n_r\bar{y}^2$$

$$V_e = \frac{\sum_{i=1}^{n_r} y_i^2 - \frac{1}{n_r}\left[\sum_{i=1}^{n_r} y_i\right]^2}{n_r - 1}$$

Equation 4

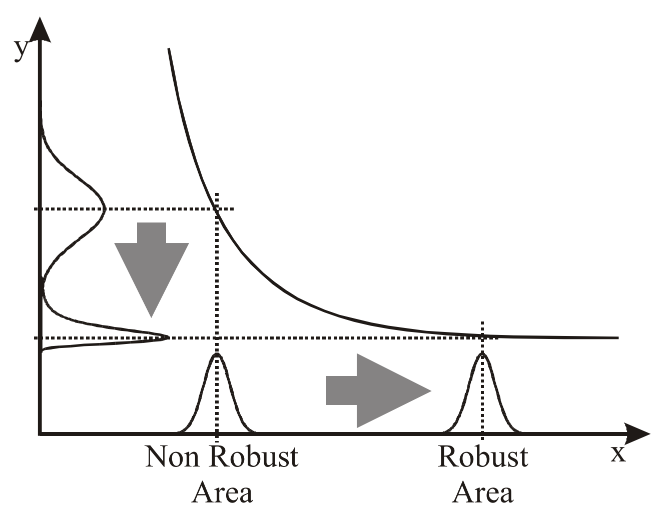


Figure 5: Robust Design by Minimizing Response Variance

This sensitivity robustness approach has the benefit of minimizing the variance in the design responses, therefore creating designs that are more consistent. However, this approach limits the design variables to insensitive areas within the possible design space. This may reduce the overall performance of the design for the sake of variance control. Also, Taguchi's signal to noise ratio may not always generate feasible and robust results as stated by (Wilde 1991). Figure 6, for example, visualizes a situation where a small variation and a nominal feasible design will generate a large number of defects compared to another response which may be more robust despite a larger variation.

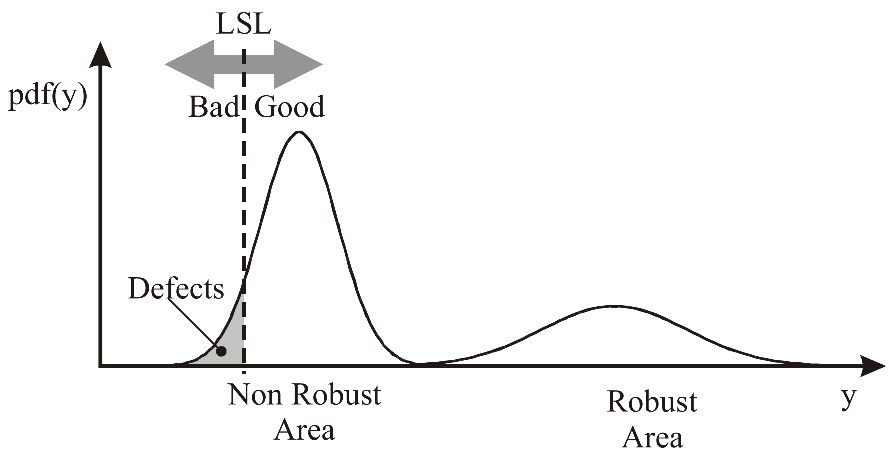


Figure 6: Robust Design by Minimizing Probability of Specification Violations

This flaw is overcome by a different approach, which is also commonly considered as robust design. (Parkinson 1995) describes this approach as feasibility robustness. The goal of this approach is to minimize the effect of the variation towards the probability of design performance satisfaction. Therefore, this approach does not minimize response variation but rather reduces the probability of violating the design specifications; i.e. the yield is maximized. (Craig 1988) utilizes this approach to optimize a robust design by maximizing the tolerance against variation. Compared to the variation minimization approach, a yield maximization approach does not restrict the design variables to insensitive areas in the design space, but rather explores the full design space.

2.3 Noise

Noise is uncontrolled and random variation during the production of the design. This noise will cause the design variables in *X* to differ for each instance of the design from the nominal values. It is possible to reduce the variation, i.e. the tolerance within which a variable can be controlled, however, this requires additional effort and cost. In addition, as implied by the Heisenberg uncertainty principle (Heisenberg 1985), it is not possible to have absolute control with a finite effort. Hence, it is impossible to completely eliminate noise in the design variables.

2.3.1 Mathematical Description

As the design responses in *Y* are a function of the design variables, these design responses *Y* also differ for each instance of the design because of the noise in the design variables *X*. The nature of the noise in the design responses *Y* depends on the nature of the noise in the design variables *X* and the underlying functional relation between the design variables *X* and the design responses *Y*.

In order to evaluate the variation in a design, this variation has to be described as a mathematical expression. There are a number of different ways to describe the noise distribution. One possible method is to define a tolerance limit with an upper and lower boundary between which a design variable or a design response varies. Another frequently utilized method is to combine these tolerance limits with a probability of the variable or response being within this limit. The process capability indices are also describing the likelihood of a variable or response being within given tolerance limit

(Kotz and Johnson 1993; Montgomery 1990; Suri et al. 1998; Tata and Thornton 1999). However, these methods do not describe the variation in sufficient detail.

In this dissertation, probability density functions will be used to describe the variation in design variables and design responses. The set of probability density functions of the design variable noises is nominated as $pdf^N(X)$, consisting of the individual probability density functions $pdf^N(x_i)$ for each design variable x_i. The set of resulting probability density functions of the design response noises is nominated as $pdf^N(Y)$, consisting of the individual probability density functions $pdf^N(y_j)$ for each design response y_j. One possible probability density function is shown in Figure 7, where the ratio of the shaded areas in the tails of the distribution to the total area of the distribution represent the percentage of parts being outside of the lower and upper specification limit *LSL* and *USL* for a design response y_j.

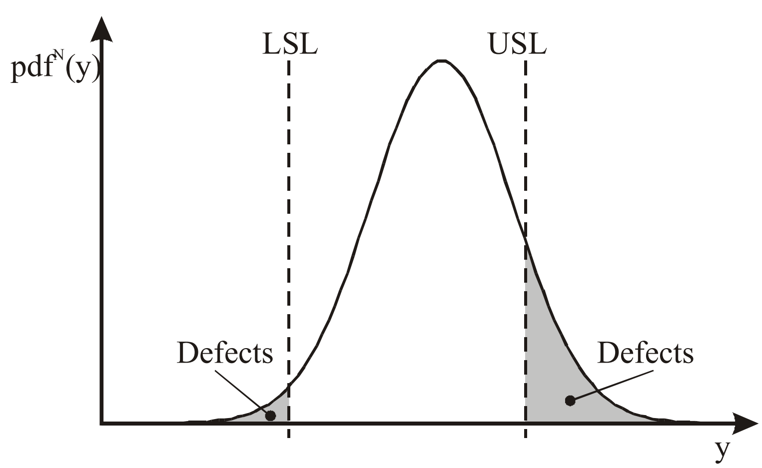


Figure 7: Probability Density Function

It is also common practice to assume standard distributions (for example normal distributions, uniform distributions, or gamma distributions) for design variables to reduce the computational effort in handling these density functions. (Siddall 1986) describes an approach to code noise in a probability density function.

2.3.2 Evaluation of Response Noise

In order to determine the effect of noise, the noise in the design responses $pdf^N(Y)$ has to be evaluated. If all design variables *X*, their noise distribution $pdf^N(X)$, and the relations are known, the response noise $pdf^N(Y)$ can be evaluated analytically as shown in Equation 5.

The form of *h* depends on the selected method for predicting the response distribution. A complete prediction of the response noise can only be done if all controlled and uncontrolled design variables are considered. A brief overview of selected methods to evaluate the design response distributions is listed in Appendix A. (Robinson 1998) also describes a number of different methods used for variation prediction.

$$pdf^N(Y) = h\left(pdf^N(X)\right)$$

Equation 5

If there is insufficient information regarding the design variable noise $pdf^N(X)$, the response noise $pdf^N(Y)$ can also be modeled according to measured data. If not all controlled and uncontrolled design variables can be considered, a prediction of the response noise will be incomplete. The predicted response distribution $pdf^N(Y)$ will be tighter than the actual response distribution due to the disregard of some noise sources. One approach is to create a model of the response noise based on measured sample data for different regions of the design space. A model predicting the response noise $pdf^N(Y)$

based on the design variable values *X* can be fitted to the experimental data using response surface methods (Myers et al. 1989).

$$pdf^{N}(Y) = h(X)$$

Equation 6

Additional variation may be introduced due to measurement errors (Dieck 1996). However, within this dissertation measurement errors are not included in the methodology. In addition, random variation in measurement errors can be reduced by repeated measurement and statistical analysis.

2.3.3 Probability of Specification Satisfaction with Noise

The probability density functions $pdf^{N}(Y)$ of the design responses *Y* can now be used to estimate the likelihood of specification satisfaction, i.e. the probability of the value of a response *y* being within the required specification limits. In the deterministic evaluation, the ability of specification satisfaction is a Boolean value, true if the response value is within the specifications or false if the response value is outside of the specification. Due to the effect of noise, however, some instances of the design might satisfy a specification while others might not satisfy a specification due to random variation. The probability of satisfying one specification and the probability of satisfying all specifications is detailed below.

2.3.3.1 Probability of Satisfying One Specification

The probability P^N_j of satisfying the specification associated with the design response y_j can be evaluated by integrating the response's probability density function $pdf^N(y_j)$ between the associated specification limits LSL_j and USL_j as shown in Equation 7. This is also visualized above in Figure 7, where the area underneath of the probability density function between the specification limits represents the probability of satisfying the given specification. For one-sided specification limits, the unused specification limit may be set to $\pm\infty$.

$$P_j^N = P\left(LSL_j \leq y_j \leq USL_j\right) = \int_{LSLj}^{USL_j} pdf^N\left(y_j\right)$$

Equation 7

2.3.3.2 Probability of Satisfying All Specifications

The previous section evaluated the probability of satisfying one specification. However, it is assumed that the quality of an instance of the design is inadequate if one or more specifications are violated. Hence, the probability of satisfying all specifications of a given instance of the design has to be evaluated. The probabilities of satisfying the individual specifications P^N_j can be combined into a joint probability P^N of satisfying all specifications, commonly described as the yield. Using Boolean notation, this probability can be expressed as shown in Equation 8. This can also be expressed as shown in Equation 9, where the product of the individual probabilities of specification satisfaction represents the probability of satisfying all specifications. The covariance is added to compensate for the effects of interactions.

$$P^N = P_1^N \cap P_2^N \cap P_3^N \cap ... \cap P_n^N$$

Equation 8

$$P^N = \prod_{j=1}^{n} P_j^N + cov$$

Equation 9

2.4 Quality Requirement

In a probabilistic design approach, a design response y_j is considered feasible if the probability of specification satisfaction $P^N{}_j$ is above a required limit P^α, also represented by a certain minimum distance α between the design response y_j and the lower and upper specification limits $LSL^N{}_j$ and $USL^N{}_j$. This distance α is measured in standard deviations $\sigma^N{}_j$ of the response noise distribution $pdf^N(y_j)$ as shown in Equation 10. A value of α of three would represent the probability P^α of at least 99.7% of the responses y_j being within the specification limits $LSL^N{}_j$ and $USL^N{}_j$ despite noise variation. Equation 11 describes the evaluation of the required standard deviation $\sigma^N{}_j$ from the noise $pdf^N(y_j)$ for normal distributions (Devore 1995). The value of P^α can be evaluated by integrating a standard normal distribution with a mean of zero and a standard deviation of one from -α to +α as shown in Equation 12. If, however, the limit P^α is given, the value of α can be calculated using the standard t distribution as shown in Equation 13.

$$LSL_j^N + \alpha \cdot \sigma_j^N \le y_j \le USL_j^N - \alpha \cdot \sigma_j^N \quad \forall j$$

Equation 10

$$\mu_j^N = \int_{-\infty}^{\infty} y_j \cdot pdf^N(y_j)dy_j$$

$$\sigma_j^N = \sqrt{\int_{-\infty}^{\infty} (y_j - \mu_j^N)^2 \cdot pdf^N(y_j)dy_j}$$

Equation 11

$$P^{\alpha} = \int_{-\alpha}^{\alpha} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$

Equation 12

$$\alpha = t_{\frac{1-P^{\alpha}}{2},\infty}$$

Equation 13

2.5 Design Optimization

Using P^{α} as the quality requirement, there may exist more than one possible feasible design within the design space. Therefore, an objective is also used to choose between different feasible designs. The objective and the subsequent optimization are described below.

2.5.1 Design Objective

Within this dissertation, the marginal part cost C^M is used to determine the objective between different feasible designs. The objective is to determine the design with the least marginal part cost C^M while still satisfying the quality requirement for all design responses *Y*. The optimization can be expressed as:

$$\begin{aligned} &min \quad C^M \\ &s.t. \quad LCL_i \le x_i \le UCL_i \quad \forall i \\ &\qquad\; LSL_j^N + \alpha \cdot \sigma_j^N \le y_j \le USL_j^N - \alpha \cdot \sigma_j^N \quad \forall j \end{aligned}$$

Equation 14

Numerous research are available in the area of objective functions. (Chen et al. 1998) develops a quality utility using compromise-programming methods. (Keeney 1974) investigates multiplicative utility functions. (Otto and Antonsson 1993) implemented the method of imprecision using fuzzy set theory. (Wilde 1991) shows the shortcomings of a signal to noise ratio and provides an improved objective.

2.5.2 Optimization Techniques

There are numerous techniques available for optimization, including functional evaluation, linear programming, gradient methods, random search patterns, genetic algorithms etc. (Cheng and Li 1997; D'Ambrosio et al. 1997; Das and Dennis 1995; Kunjur and Krishnamurty 1997; Osyczka 1985; Steuer 1986) describe different multiattribute optimization approaches. (Kazmer et al. 1996; Maglaras et al. 1996) also provide probabilistic optimization techniques. For a detailed discussion of a variety of different optimization techniques please refer to (Reklaitis et al. 1983). Optimization techniques are well developed and can be used as needed. Therefore, they will not be described in further detail in this dissertation.

2.6 Example: I-Beam

Throughout the dissertation, a simple example of an extruded cast iron I-beam is used to demonstrate the methodology. The beam is specified by the beam height and the elastic modulus of the material. The control limits for these design variables are given in Table 1. Although the material type is a discrete variable, it is considered continuous to simplify the example. Other design variables are set to a fixed value or described in dependence with the height and modulus. The deflection of the beam is the specified response. In addition, the part cost of the beam is evaluated. An overview of the design responses is shown in Table 2. The relation between the design variables and the design response and the cost are determined using an analytical model, which is then simplified to a response surface method using a design of experiments. Note that the stress of the beam is always assumed to be less than the critical stress. A detailed description can be found in Appendix B. In addition, a more complex real world example will be used to demonstrate the developed method in Chapter 6.

Table 1: I-Beam Design Variables

Design Variable	Nom.	Unit	LCL	UCL
Beam Height	H	mm	30	60
Elastic Modulus	E	N/mm^2	90,000	185,000

Table 2: I-Beam Design Responses

Design Response	Nom.	Unit	LSL	USL
Deflection	D	mm	n/a	0.3
Part Cost	C^{Part}	$	n/a	n/a

2.6.1 Physical Model

This example is an extruded I-beam with a constant cross section. The beam has a fixed support at one end and is loaded with a force at the other end. The system is assumed to be static. The I-beam is shown from the side in Figure 8 and as a cross section in Figure 9. The nomenclature of the drawings is explained in Table 3.

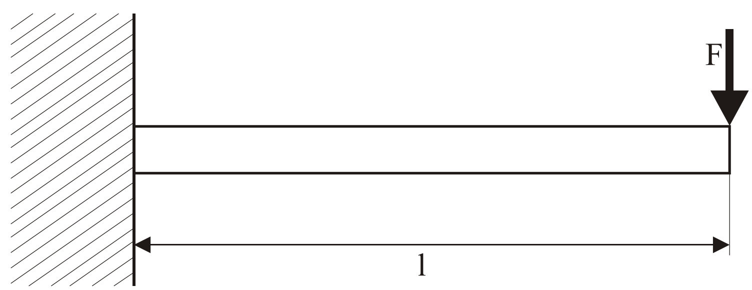


Figure 8: I-Beam Side View

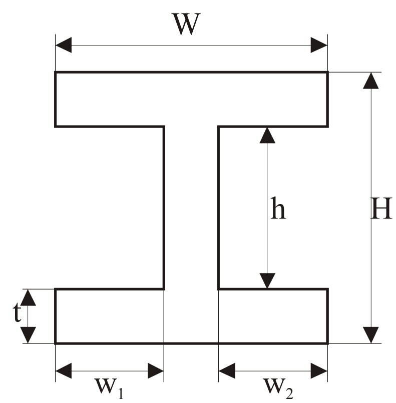


Figure 9: I-Beam Cross-Section

Table 3: I-Beam Drawing Nomenclature

Design Variable	Symbol
Applied Load	F
Beam Length	l
Beam Width	W
Web Thickness	w_1, w_2
Beam Height	H
Web Height	h
Wall Thickness	t

There are two different responses evaluated for the given beam example. These responses are the deflection of the beam and the cost of a single beam. The deflection is constrained to be less than an upper deflection limit, whereas the part cost is unconstrained.

2.6.1.1 Deflection

The deflection is evaluated using the section modulus. Using standard equations from (Beitz and Küttner 1995), the moment of inertia I_y is evaluated as shown in Equation 15. The underlying relations for the beam geometry are shown in Equation 16. The deflection D is evaluated in Equation 17, where E is the elastic modulus of the material.

$$I_y = \frac{W \cdot H^3 - w \cdot h^3}{12}$$

Equation 15

$$w = w_1 + w_2 = W - t$$
$$h = H - 2 \cdot t$$

Equation 16

$$D = \frac{F \cdot l^3}{3 \cdot E \cdot I_y}$$

Equation 17

2.6.1.2 Part Cost

For this example, the part cost is a function of the volume of the part and the cost of the material. Fixed production costs are ignored for the sake of simplification. The volume V can be calculated as shown in Equation 18. Equation 19 evaluates the marginal part cost C^{Part} using the volume V, the density ρ and the cost of the raw material C^R.

$$V = [W \cdot H - (w_1 + w_2) \cdot h] \cdot l$$

Equation 18

$$C^{Part} = V \cdot \rho \cdot C^{R}$$

Equation 19

2.6.1.3 Simplifying Assumptions

The above example is simple, yet already requires numerous design variables to evaluate the design response. In order to reduce the complexity, some variables are assumed to be constant for the simplified example. The beam length l is set to 1000mm and the load F is assumed to be constant at 100N. The beam is assumed to have a constant width W of 30mm. The wall thickness t is also assumed to be constant at 5mm. Therefore, the deflection D from Equation 17 can be simplified as shown in Equation 20.

$$D = \frac{80{,}000{,}000{,}000 \cdot \left[\frac{mm^2}{N}\right]}{E \cdot \left[-5 \cdot (-10[mm] + H)^3 + 6 \cdot H^3\right]}$$

Equation 20

The cost of the part C^{Part} depends on the density of the material ρ and the cost of the raw material C^{R}. For this example, the density ρ is assumed to be constant at 7,200 kg/m^3. The cost of the raw material varies with the modulus of the material. Within this example, the cost of the cast iron increases linearly from \$500 pert ton at the lower constraint limit of the modulus to \$540 per ton at the upper constraint limit of the modulus as shown in Equation 21. Combining all assumptions and simplifications into one equation evaluates the part cost as shown in Equation 22.

$$C^R = \frac{1{,}097{,}500 \cdot \left[\frac{N}{mm^2}\right] + E}{2{,}375{,}000 \cdot \left[\frac{N}{mm^2 \cdot \$}\right]}$$

Equation 21

$$C^{Part} = \frac{9 \cdot \left(1{,}097{,}500 \cdot \left[\frac{N}{mm^2}\right] + E\right) \cdot \left(50[mm] + H\right)}{593{,}750{,}000} \cdot \left[\frac{\$ \cdot mm}{N}\right]$$

Equation 22

2.6.1.4 Design Variable Noise

The prediction of the design responses' noise variation is required. Within this example, noise is assumed to occur in the beam height H and the modulus E, having a standard normal distribution. The noise deviation is σ_H and σ_E for the beam height H and the modulus E respectively. Using a moment matching method the resulting noise deviation σ_D of the deflection D can be evaluated as shown in Equation 23.

$$\sigma_D = \sqrt{\left(\frac{dD}{dH} \cdot \sigma_H\right)^2 + \left(\frac{dD}{dE} \cdot \sigma_E\right)^2}$$

Equation 23

According to (Beitz and Küttner 1995), the tolerance limits of the extrusion process are typically about ±0.8%. If the tolerance limit is assumed to span three standard deviations, then the standard deviation of the beam height is approximately 0.133 mm for a beam height of 50mm. Therefore, the standard deviation of the beam height is set to 0.1 mm within this example. The modulus E is also assumed to vary. Within this example, the influence of the temperature is used to determine the standard deviation of the modulus. Between room temperature and an upper operating temperature limit of 200C, the modulus of cast iron and similar materials is reduced by up to 5,000N/mm^2 according to (Beitz and Küttner 1995). Assuming a temperature deviation of ±70C around room temperature, a standard deviation σ_E of the modulus E of 1,700 N/mm^2 can be estimated. These noise variations of the design variables are used to evaluate the noise variation of the design response in Equation 23.

2.6.1.5 Physical Model Summary

This section is a summary of the simplified physical model as described above. There exist two design variables, the beam height H and the elastic modulus E as shown in Table 4. These variables determine two design responses, the deflection D and the marginal part cost C^M, as shown in Table 5. The functional relations are shown previously in Equation 20 for the deflection D, Equation 23 for the standard deviation σ_D of the deflection D, and Equation 22 for the marginal part cost C^{Part}.

Table 4: I-Beam Design Variables Physical Model Summary

Design Variable	Nom.	Unit	LCL	UCL	Noise Deviation
Beam Height	H	mm	30	60	0.1
Elastic Modulus	E	N/mm^2	90,000	185,000	1,700

Table 5: I-Beam Design Responses Physical Model Summary

Design Response	Nom.	Unit	LSL	USL
Deflection	D	mm	n/a	3
Part Cost	C^{Part}	$	n/a	n/a

2.6.2 Response Surface Model

The above example is fairly straightforward. However, engineering design frequently involves complex examples, where the evaluation of one design requires significant effort by computational simulations or physical experiments. In this case, response surface methods are frequently used to model the relations in order to simplify the evaluation effort. Although this would not be necessary for a simple example as described above, a response surface model will be created to show the resulting model uncertainties. These model uncertainties will be significant for the flexible design methodology described in Chapters 3 and 4.

2.6.2.1 Design of Experiments

A central composite design of experiments was selected in order to create a second order response surface (Schmidt and Launsby 1994). The sample points are shown graphically in Figure 10, where the circles represent sample points taken. The axis scales are given both as coded between ±1 and as physical values. An overview of the resulting data is shown in Table 6.

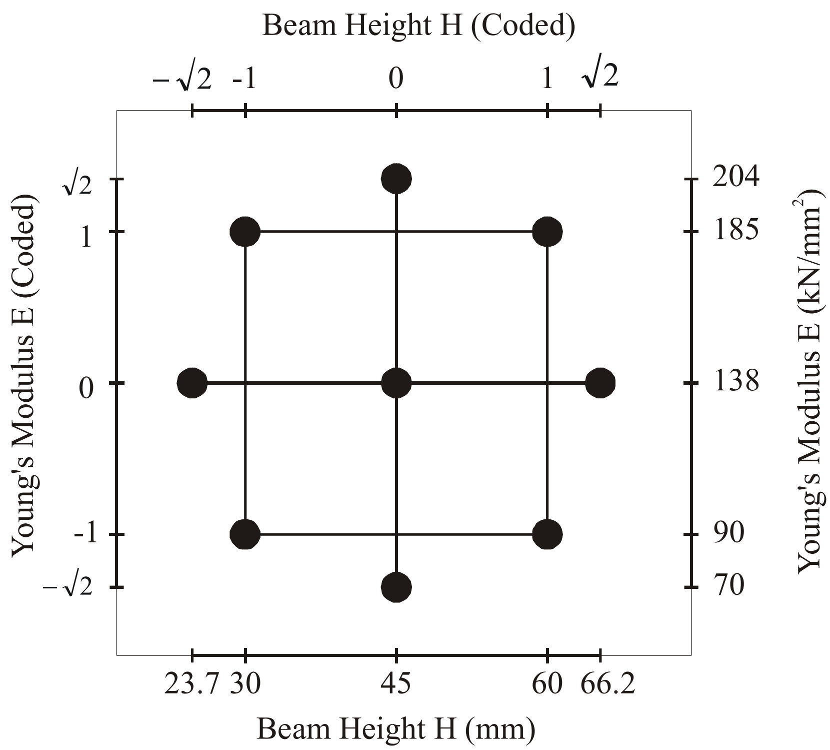


Figure 10: Central Composite Design of Experiments

Table 6: Design of Experiments Data

Run	H	E	D	σ_D	C^{Part}
1	30	90,000	7.28	0.151	1.44
2	30	185,000	3.54	0.044	1.56
3	60	90,000	1.33	0.026	1.98
4	60	185,000	0.64	0.006	2.14
5	45	70,324	3.42	0.085	1.68
6	45	204,675	1.18	0.012	1.88
7	23.7	137,500	8.60	0.141	1.38
8	66.2	137,500	0.68	0.009	2.18
9	45	137,500	1.75	0.024	1.78

2.6.2.2 Resulting Model

In order to distinguish the response surface prediction model from the actual physical model, the nomenclature for the response surface model uses the notation of *X* and *Y* as used for the description of the flexible design methodology. The design variable and the resign response nomenclature are shown in Table 7 and Table 8.

Table 7: I-Beam Response Model Design Variables

Design Variable	Nom.	Unit	LCL	UCL
Beam Height	x_1	mm	30	60
Elastic Modulus	x_2	N/mm^2	90,000	185,000

Table 8: I-Beam Response Model Design Responses

Design Response	Nom.	Unit	LSL	USL
Deflection	y_1	mm	n/a	3
Deflection Deviation	$\sigma^N{}_1$	mm	n/a	n/a
Marginal Part Cost	C^M	$	n/a	n/a

In order to determine the model equations, the data in Table 6 has to be augmented to include columns for the constant x_1 and x_2 values, the interaction x_1*x_2, and the squared effects x_1^2 and x_2^2. This is shown schematically in Equation 24, where the second and third column on the matrix *LHS* represents the variable values from the *H* and *E* columns of Table 6. The other columns are a function of the second and third column.

$$LHS = \begin{bmatrix} \bar{1} & \overline{x_1} & \overline{x_2} & \overline{x_1} \cdot \overline{x_2} & \overline{x_1^{\;2}} & \overline{x_2^{\;2}} \end{bmatrix}$$

Equation 24

The above data now forms Equation 25, where *LHS* represents the left hand side of the equation, i.e. the design variable data including interactions as shown in Equation 24, *RHS* represents the right hand side of the equation, i.e. the measured design responses from the last three columns of Table 6, and *M* will contain the model parameters. This matrix equation can be solved as shown in Equation 26, using the transpose and inverse of the design variable matrix *LHS*. Due to prediction errors, the standard deviation of the deflection can fall below zero. To avoid negative deviations the prediction of the deviation is limited to values above the smallest deviation in the design of experiments.

The resulting prediction equations for the deflection y_1, the standard deviation of the noise $\sigma^N{}_1$ of the deflection y_1 and the marginal part cost C^M are shown in Equation 27, Equation 28, and Equation 29. The regression coefficients indicate a good fit.

$$LHS \cdot M = RHS$$

Equation 25

$$M = \left(LHS^T \cdot LHS\right)^{-1} \cdot LHS^T \cdot RHS$$

Equation 26

$$\begin{aligned} y_1 = & \\ & 32.8 - 0.87 \cdot x_1 - 9.35 \cdot 10^{-5} \cdot x_2 + \\ & \quad + 1.07 \cdot 10^{-6} \cdot x_1 \cdot x_2 + 6.12 \cdot 10^{-3} \cdot x_1^2 + 9.16 \cdot 10^{-11} \cdot x_2^2 \quad R^2 = 0.9935 \end{aligned}$$

Equation 27

$$\begin{aligned} \sigma_1^N = Max[0.0065,\ & 0.74 - 0.017 \cdot x_1 - 3.33 \cdot 10^{-6} \cdot x_2 + \\ & + 3.06 \cdot 10^{-8} \cdot x_1 \cdot x_2 + 1.09 \cdot 10^{-4} \cdot x_1^2 + 4.89 \cdot 10^{-12} \cdot x_2^2] \quad R^2 = 0.996 \end{aligned}$$

Equation 28

$$\begin{aligned} C^M = & 0.83 + 0.016 \cdot x_1 + 7.57 \cdot 10^{-7} \cdot x_2 + \\ & + 1.51 \cdot 10^{-8} \cdot x_1 \cdot x_2 - 1.12 \cdot 10^{-17} \cdot x_1^2 + 7.80 \cdot 10^{-24} \cdot x_2^2 \quad R^2 = 1.00 \end{aligned}$$

Equation 29

2.6.2.3 Model Validation

The response surface method tries to replicate the set of data points with a second order response surface. As there are nine data points, but only six response surface parameters, it is not guaranteed to create a perfect match. Instead, the response surface tries to minimize the sum of the squares of deviations from the data points (Wolfram 1996). Therefore, the prediction model will not necessarily predict the sample points exactly, and exhibit a prediction error. Furthermore, the response surface model was created using selected data points from the design space. Therefore, the response surface may not follow the real design space between the sample points. In the I-beam example, the quadratic response surface cannot model an inverse cubed function as the deflection shown in Equation 20.

However, in most response surface applications the prediction error is measured, but rarely included in the design evaluation. Therefore, within this Chapter 2 the model is only validated and the prediction error distribution estimated. The effect of the prediction error on the design will be discussed in Chapter 3 and a possible solution using the flexible design methodology will be presented in Chapter 4.

The validation of the model is performed by comparing the prediction model with the physical model for a limited number of random sample points. The response surface model of the deflection y_1 has been compared with the analytical model of the deflection D using 10 randomly distributed points within the design space as shown in Equation 30. This is to simulate a typical validation approach, where the model is compared to a limited number of sample points. The use of the data from the experimental design is usually avoided to estimate the prediction error between the sample data. The estimation of the mean error is shown in Equation 31, where n represents the number of sample points.

$$e(y_1) = y_1 - D_E$$

Equation 30

$$\overline{e(y_1)} = \frac{\sum e(y_1)}{n}$$

Equation 31

$$\sigma(e(y_1)) = \sqrt{\frac{\sum \left(e(y_1) - \overline{e(y_1)}\right)^2}{n-1}}$$

Equation 32

Equation 32 shows the calculation of the standard deviation of the error for the deflection. The mean and the standard deviation for the error in the noise prediction and the marginal cost prediction have also been evaluated in a similar way and are shown in Table 9. Figure 11 shows a graph comparing the response surface model prediction with the actual physical model. The top graph varies the beam height x_1, while keeping the modulus x_2 constant at 137,500N/mm^2. The bottom graph varies the modulus x_2, while keeping the beam height x_1 constant at 45 mm.

Table 9: Prediction Error Mean and Deviation

Prediction	Nom.	Mean Error	Error Standard Deviation
Deflection	y_1	0.4	0.45

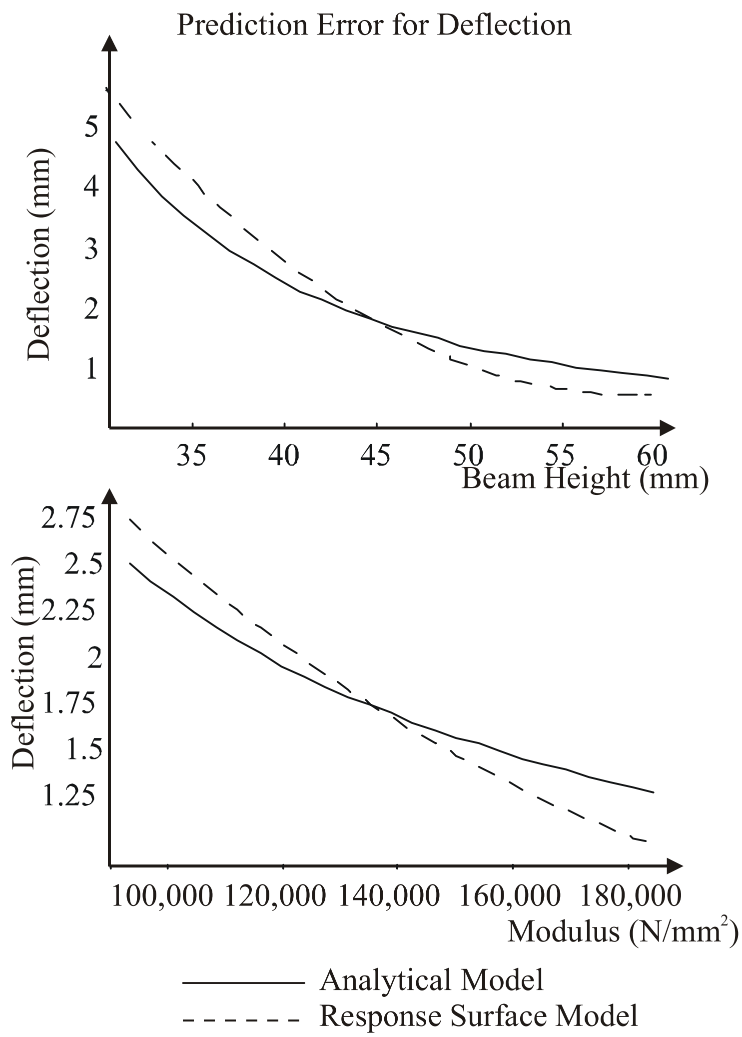


Figure 11: Prediction Error for Deflection

It can be seen that the error of the prediction model can make a significant difference regarding the development of the design. The effect of these uncertainties will be discussed in more detail in Chapter 3, and the handling of these uncertainties will be described as part of the flexible design methodology in Chapter 4. As for now, the prediction error is ignored, and the prediction models are assumed to be correct.

2.6.3 I-Beam Robust Design

The quality requirement for the I-Beam is to have at least a distance of α standard deviations between the mean response y_1 of the deflection and the upper specification limit $USL^N{}_1$. Within this example α is required to be at least three, representing three standard deviations distance between the mean response and the upper specification limit. The upper specification limit is 3mm, requiring the deflection to be below the given limit. The two design variables have to be within the given constraint limits as shown in Table 4. The optimization objective is shown in Equation 33.

$$\begin{aligned} min \quad C^M \quad s.t. \quad & y_1 \le 3.0 - 3 \cdot \sigma_1^N \\ & 30 \le x_1 \le 60 \\ & 90{,}000 \le x_2 \le 185{,}000 \end{aligned}$$

Equation 33

Using a gradient search method, the optimal beam design was determined. As shown in Table 10, the optimal design uses a modulus of 182,150N/mm^2 and reduces the beam height to 34.8mm in order to generate a lowest marginal part cost of $1.64.

Table 10: Optimal Beam Design

Variable	Nom.	Value
Beam Height	x_1	34.8
Modulus	x_2	182,150
Response	Nom.	Value
Deflection	y_1	2.89
Marginal Cost	C^M	1.64

2.7 Conclusion

The method of robust design is a standard engineering procedure. The design variables including noise are evaluated using transfer functions to estimate probabilistic design responses. Then these probabilistic design responses are used to develop an optimal design that is likely to meet specifications.

However, the prediction of the design responses is only as accurate as the underlying transfer functions and design variables. Inaccuracies in the transfer functions or uncertainties in the design variables will cause uncertainty in the design responses. In this case, a design optimized for robustness might generate excessive defects and therefore excessive cost. In the next chapters, methods are discussed to facilitate development of robust designs considering not only noise but also uncertainties.

CHAPTER 3

DESIGN UNCERTAINTY

3.1 Introduction

This chapter discusses the sources and effects of uncertainty in design. Uncertainty is defined differently by different researchers. (Finch and Ward 1997) use the term uncertainty for both recurring noise variation and one-time offsets in the design in a set-based robust design method. (Antonsson and Otto 1995; Wood and Antonsson 1990) model uncertainty in design variables during the early design stages, which they describe as imprecision, whereas their definition of uncertainty describes random variation in the design environment. (Nikolaidis et al. 1999) uses the term "uncertainty" both for random variation and for modeling uncertainty. (Sarbacker 1998) defines uncertainty as the lack of information and evaluates the effect of uncertainty on the risks of the design process. (Cohen and Freeman 1996) describe a collection of ideas regarding uncertainty in decision making for naval anti air warfare. (Blackmond-Laskey 1996) analyzes uncertainty in the model structure and presents approaches for treating and reducing model uncertainty. (Lehner et al. 1996) tries to clarify the understanding of uncertainty and higher order uncertainty, i.e. uncertainty about uncertainty. (Otto and Wood 1995) analyze uncertainty in the selection of a design concept and describes a method to quantify the uncertainty. (Plunkett and Dale 1988) categorizes and discusses different design models with respect to the real design to improve the trade-off between quality and cost.

This dissertation defines uncertainty as the probabilistic relation between the model response and the actual response. Prediction uncertainty is often overlooked when creating a design. The design is frequently optimized according to the design objective using the prediction models under the assumption of these models being accurate. However, the effects of uncertainty might jeopardize the finely adjusted "*optimal*" design, causing excessive defects and requiring design changes. The causes of uncertainty are described in detail below, with an additional emphasis on the creation of a prediction model from a finite number of data points. This chapter discusses only the causes and effects of uncertainty, but does not present an efficient solution to avoid the negative effects of uncertainty. A proposed solution, the flexible design methodology, will be presented in Chapter 4.

3.2 Sources of Prediction Uncertainty

Three main sources of uncertainty have been identified. The first source of uncertainty originates from assumptions, where the information is not known but assumed. The second source is simplification, where the information is available but not used. The third source of uncertainty is error, where the available information is incorrect or used incorrectly. These sources will be explained in more detail below. Note that an exact distinction between the sources of uncertainty is not always possible. Also, note that within the scope of this dissertation, only prediction uncertainty in the engineering design is considered. Other sources of uncertainty are mentioned, but not discussed in detail.

3.2.1 Assumptions: Lack of Information

An assumption is information generated without sufficient proof of correctness. Assumptions are made when information is created not based on other information. In the creation of a prediction model for engineering design, not all required information might be known. In order to generate a prediction model, information is often assumed, based frequently on human experience and educated guesses. Often, it might be impossible to create a prediction model without assumptions, in which case a moderately inaccurate model is preferred over no model at all. The assumptions might be incomplete or wrong, subsequently reducing the accuracy of the prediction model. If reasonable assumptions are used in moderation, the resulting uncertainty might be justified on economic grounds compared to the potentially significant effort of proving an assumption valid.

In fluid dynamics, for example, flows are frequently assumed to be laminar or turbulent depending on the Reynolds number, yet the exact behavior of the fluid is unknown. Although the linear or turbulent nature of flow can be investigated in more detail, the effort frequently exceeds the value of the gained accuracy. Another frequent assumption in probabilistic design is the assumption of probability distributions to be standard normal. However, statistical validation is typically expensive and time consuming. Frequently, the nature of the distribution is ignored completely, with first and second moments calculated for a finite sample, according to a normal distribution. Another example is the finite element method (Burnett 1988), in which assumptions are made about the geometric modeling, numerical truncation, material constitutive equations, initial and loading conditions, and physical laws.

3.2.2 Simplifications: Disregard of Known Information

Simplification is the disregard of known information. In the creation of a prediction model for engineering design, information might be available yet not included in the prediction model. Such simplifications are typically used in order to reduce the complexity of the model, as for example in the finite element method loads are often assumed to be point loads or uniform load distributions, although it is known that this is not the case in reality. However, the small loss of accuracy for these simplifications is often accepted compared to the effort of modeling the loads in more detail. (Alvin et al. 1998) describes a method to quantify uncertainty in structural dynamics, also distinguishing between uncertainty and error. Another example is the I-beam presented in Chapter 2. The physical model calculates the deflection based on an I-beam cross-section with perfect right angles. However, the actual beam will have rounded edges, with known standard geometries. Yet, the small resulting inaccuracies of the model are commonly accepted compared to the effort of integrating across a more complex section. Another example is the simplification of variation. A randomly distributed value is often simplified to a deterministic value in order to reduce the model complexity. This deterministic value might be the mean of a sample, e.g. for the melt temperature of an injection molding process, or it might be an upper or lower limit on a sample, e.g. the largest occurring load a structure is expected to withstand. This simplification creates inaccuracies, yet the resulting inaccuracies are accepted compared to the effort of incorporating distributed variables into a model.

3.2.3 Errors: Wrong Information

While simplifications and assumptions are useful in engineering to reduce the engineering effort, errors are generally undesirable. The effect of incorrect information and the incorrect use of information during the creation of a prediction model may have drastic effects on the accuracy of the model. Depending on the type of error, the model uncertainty can range from slightly inaccurate to completely wrong.

A prime example of human error is the Mars Climate Orbiter (Pollack 1999). The prediction model for the location and course correction of the spacecraft contained errors due to the confusion of the unit system. As a result, the craft entered the atmosphere of the mars at a too low altitude and was lost. Other errors occur frequently in the calculation and programming of prediction models, where for example, a minus sign might be forgotten in the prediction equation or prediction software might have a programming bug due to a division by zero, which occurs only under certain conditions. Errors also can come from faulty data, for example, when a material property in a data table does not represent the actual material property.

3.2.4 Other Causes of Uncertainty

This dissertation considers only prediction model uncertainty. However, there are numerous other sources of uncertainty in an engineering design. These include, for example, uncertainty regarding the use of the product by the end user. Specifications are used to define the boundaries of possible uses; however, these boundaries may also be inaccurate. Thus, the specifications are subject to uncertainty. Other sources of uncertainty are based on the market environment, as for example the demand, competition, exchange rates, or interest rates that will affect the commercial success and the profitability of a design. (Jordan and Graves 1995) for example investigates the effect of market uncertainties on the manufacturing process. However, these sources of uncertainty will not be considered within the scope of this dissertation.

3.3 Characterization of Uncertainty

3.3.1 Description of Uncertainty

Uncertainty has to be mathematically described to develop a flexible design methodology. Different approaches have been found in literature. (Goodwin et al. 1990) for example uses Bayesian networks in the description of model uncertainty. (Otto and Wood 1995) use a degree of confidence on the prediction accuracy to measure uncertainty. (Atwood and Engelhardt 1996) also analyze uncertainty by using prediction intervals and confidence intervals. (Maciejewski 1997) expands the idea of uncertainty intervals by using asymmetric model coefficients.

The description of a prediction error is central to the description of the uncertainty. Within this dissertation, the prediction errors E of the design responses Y is formulated using a probability density function $pdf(E)$ similar to the formulation of the noise as described in section 2.3. (Petkov and Maranas 1997) also uses probability function for the modeling of uncertainty in chemical prediction models. The probability density functions $pdf(E)$ for the prediction errors E used within this dissertation consists of the individual probability density functions $pdf(e_j)$ of the error e_j of each design response y_j. The prediction error is the difference between the mean predicted response y_j and the actual response y^*_j as shown in Equation 34. If the prediction error e_j is not known exactly, but only as a distribution *pdf(e_j)*, the actual response can also only be described as an uncertainty distribution *$pdf^U(y_j)$*. This uncertainty distribution is the joint distribution of the prediction error distribution and the dirac delta function *$\delta(y_j)$* as shown in Equation 35 and visualized in Figure 12.

$$y_j^* = y_j + e_j$$

Equation 34

$$pdf^U(y_j) = h[\delta(y_j), pdf(e_j)]$$
$$\text{where} \quad \delta(y_j) = 0 \ \forall y_j \neq \mu_j^N$$
$$\int_{-\infty}^{\infty} \delta(y_j) dy = 1$$

Equation 35

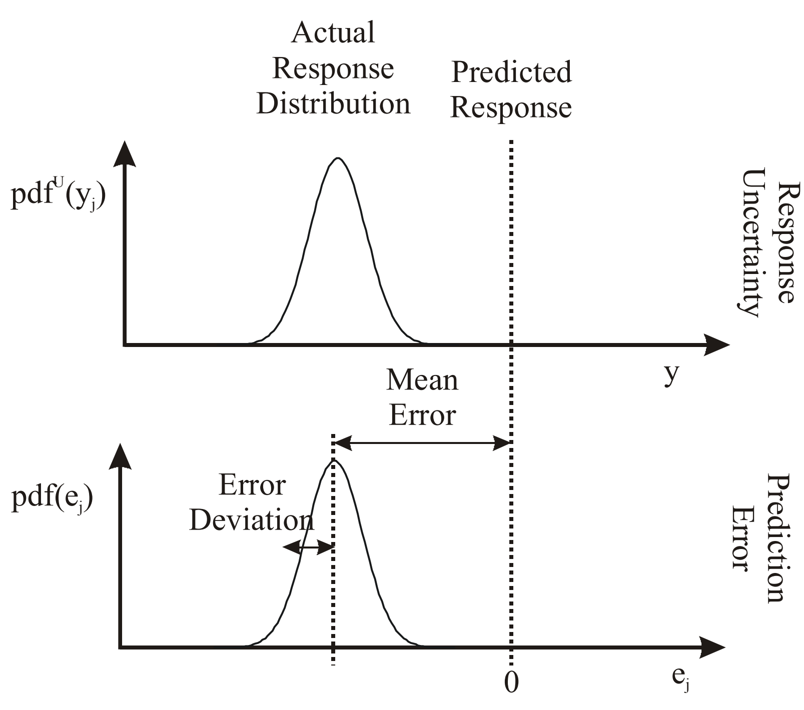


Figure 12: Predicted Response and Actual Response Distribution

3.3.2 Measurement of the Prediction Error

The uncertainty of the prediction model consists of the mean prediction error and the error distribution as shown above in Equation 35 and visualized in Figure 12. Therefore, the distribution $pdf(e_j)$ of the prediction error e_j is required in order to calculate the response distribution under uncertainty $pdf^U(y_j)$. However, this prediction error is difficult to estimate analytically. The utilized approach is to evaluate the prediction model for a number of samples for which the actual response is known. The actual response can be measured from historic data, physical experiments, or by evaluating a trusted prediction model. A simple stress model for example can be compared to a more accurate finite element analysis or experimental investigation for a given number of sample points. If an interpolation or extrapolation was used to create the prediction model, it is recommended not to reuse the data points for the model creation, i.e. the validation of the model should utilize different data to avoid biasing the prediction uncertainty. Information regarding interpolations and extrapolations can be found in Appendix C.

Based on the set of prediction errors for the sample points, a prediction error distribution can be created. Standard probability distributions are frequently assumed. The goodness of the fit of the distribution on the data can be determined if a sufficiently large sample size exists (Devore 1995). If no goodness of fit is evaluated, a distribution has to be assumed. (Myers and Montgomery 1995) describes the creation of confidence intervals in response surface models using the estimated standard deviation of the response and the standard t distribution as shown in Equation 36.

$$\mu^Y - t_{\frac{\alpha}{2},n-p} \cdot \sigma^Y \le \mu^Y \le \mu^Y + t_{\frac{\alpha}{2},n-p} \cdot \sigma^Y$$

Equation 36

3.3.3 Consequences of Uncertainty

The distribution of the design responses under uncertainty $pdf^U(Y)$ has to be evaluated for all design responses Y. This response uncertainty may cause the actual design to violate the quality requirements as defined in section 2.4. A design satisfies the quality requirement if the mean response y_j is at least a certain number of standard deviations away from the closest specification limit LSL^N_j or USL^N_j using a similar approach as described by (Myers and Montgomery 1995). The number of deviations is described as α and depends on the required percentage of good parts P^α. This can be represented as specification limits for the design under uncertainty LSL^U_j and USL^U_j. The calculation of these limits is shown in Equation 37 as derived from the quality requirement in section 2.4. The quality requirement is satisfied if Equation 38 is true.

$$LSL_j^U = LSL_j^N + \alpha \cdot \sigma_j^N \quad \forall j$$
$$USL_j^U = USL_j^N - \alpha \cdot \sigma_j^N \quad \forall j$$

Equation 37

$$LSL_j^U \le y_j \le USL_j^U \quad \forall j$$

Equation 38

Therefore, the probability $P^L{}_j$ and $P^U{}_j$ of violating the upper or lower specification limit $LSL^U{}_j$ and $USL^U{}_j$ under uncertainty can be calculated as shown in Equation 39 and Equation 40. These probabilities are different from the probabilities used for robust design since 1) the uncertainty distributions differ from the noise distributions, and 2) the specification limits are tighter. A graphical representation is shown in Figure 13, where the predicted response *y* satisfies the quality requirement, yet the uncertainty distribution has a 15% chance of violating the lower specification limit $LSL^U{}_j$ under uncertainty, i.e. a 15% chance that the yield is too small..

$$P_j^L = \int_{-\infty}^{LSL_j^U} pdf^U(y_j)dj \quad \forall j$$

Equation 39

$$P_j^U = \int_{USL_j^U}^{\infty} pdf^U(y_j)dj \quad \forall j$$

Equation 40

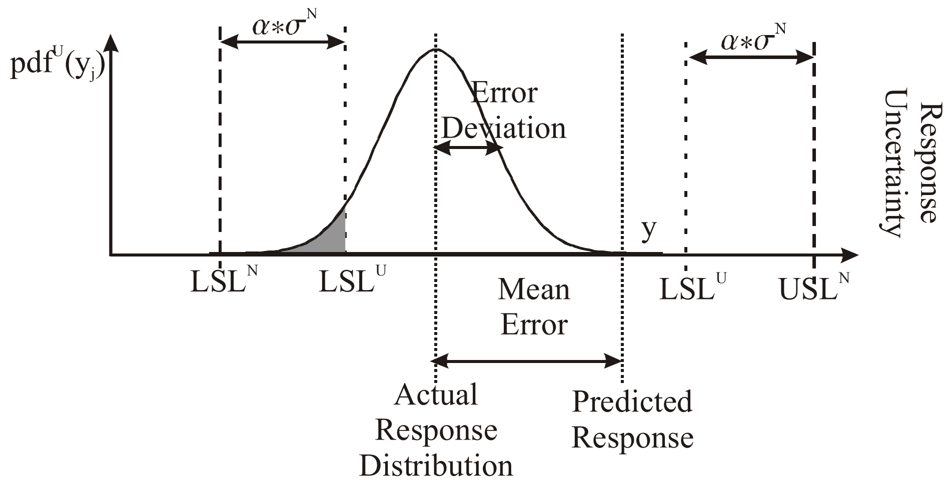


Figure 13: Probability of Violating the Quality Requirement

It is possible to estimate the joint probability of satisfying all quality requirements under uncertainty. With respect to the nomenclature used in the next chapter, this probability is nominated as $P^M{}_1$, representing the expected outcome of not violating any quality requirement. This probability can be evaluated as shown in Equation 41, where the joint probability of no specification violation with respect to interactions is estimated.

$$P_1^M = \prod_{j=1}^{n}\left[1-\left(P_j^U + P_j^L\right)\right] + \mathrm{cov}$$

Equation 41

Although it is possible to optimize the design in order to reduce the probability of specification violation under uncertainty, this is not advisable. The noise distributions $pdf^N(Y)$ and the uncertainty distributions $pdf^U(Y)$ differ significantly. Noise variation causes every instance of a design to differ from every other instance of a design and cannot be adjusted for. Therefore, the design has to be robust against noise, as described by the quality requirement. However, it is possible to adjust for prediction uncertainties, as this is a one-time error between the predicted response and the actual mean response of the design. Therefore, a design does not have to be robust against uncertainty as it is possible to change the design in order to compensate for prediction errors. It is not necessary to improve the robustness of the design for uncertainty - as this frequently increases the cost of the product – if the design is flexible enough to adjust for the prediction uncertainty. The flexible design methodology described in the next chapter enables the design team to improve the design flexibility in order to reduce the overall expected cost including the cost of possible changes.

3.4 Example: I-Beam

The analysis of the effects of uncertainty will be demonstrated on the I-beam example introduced in Chapter 2. First, the sources of uncertainty will be determined and their magnitude estimated. The handling of the uncertainty will then be described according to the above method.

3.4.1 Sources of Uncertainty

There are two sources of possible prediction errors influencing the probability density function $pdf(e_1)$ of the combined prediction error e_1 of the deflection y_1. First, there is uncertainty due to the fitting of a response surface onto a set of data points. This error was mentioned in Chapter 2. Secondly, there are some uncertainties due to some assumptions of the physical model used in Chapter 2. Both sources are described in detail below.

3.4.1.1 Response Surface Model Errors

The response surface method tries to match a second order response surface with six parameters to nine data points by minimizing the sum of the squares of deviations from the data points (Wolfram 1996). Therefore, the prediction model will not necessarily predict the sample points exactly, but rather have a prediction error and may not follow the real design space between the sample points. In Chapter 2, the prediction models have been compared with the actual models using 10 random sample points within the design space. For each sample point, the error of the deflection $e(y_1)$ was determined, and the mean and standard deviation of the error was estimated as shown in Table 11.

Table 11: Prediction Error Mean and Deviation

Prediction	Nom.	Mean Error	Error Standard Deviation
Deflection	y_1	0.4	0.45

Within this dissertation, the error distribution of the deflection is assumed to be normal distributed. A closer investigation reveals that the actual true mean error e_1 for the deflection y_1 is 0.1184mm and the standard deviation is 0.452mm, using Equation 42 and Equation 43.

$$\mu_1^E = \int_{LCL_1}^{UCL_1} \int_{LCL_2}^{UCL_2} e_1 \cdot P(e_1) dx_1 dx_2$$

Equation 42

$$\sigma_1^E = \int_{LCL_1}^{UCL_1} \int_{LCL_2}^{UCL_2} \left(e_1 - \mu_1^E\right)^2 \cdot P(e_1) dx_1 dx_2$$

Equation 43

In addition, the error distribution cannot be assumed to be standard normal distributed. Figure 14 shows a histogram of the prediction error for 1000 random samples. It can be seen clearly that the prediction error is not standard normal distributed. However, in order to simplify the calculations for the simple example, a normal distribution is used to describe the prediction error probability density function.

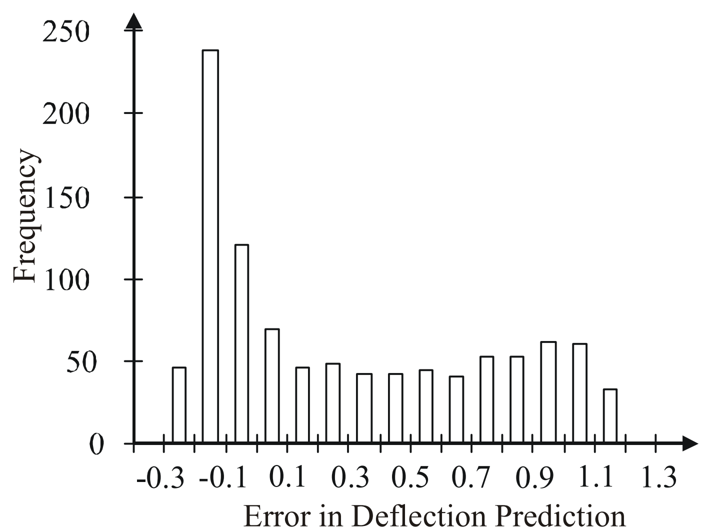


Figure 14: Prediction Error Histogram for 1000 Samples

3.4.1.2 Physical Model Assumptions

However, there exist also some assumptions regarding the physical model as described in Chapter 2. For example, the cross section of the I-beam is assumed to have all corners at right angles. However, an extruded beam would most likely have rounded corners or chamfers. Rounded edges are easier to manufacture, and also they avoid unnecessary stress concentrations. The exact moment of inertia can be evaluated by integrating the distance from the centerline over the cross sectional area.

In addition, the beam may be modified in order to attach the beam to the fixed support or to attach the load to the beam. Depending on the attachment type the stiffness of the beam may be increased due to welding additional material on the beam or may be weakened due to holes drilled for attachments, and the deflection of the beam may change. However, as neither the information regarding the rounded edges nor the information of the modifications is detailed in the physical example, the deflection predicted in the physical model will differ from the actual deflection occurring in a real beam. This deflection error is also estimated to be normal distributed with a mean of zero and a standard deviation of 0.05mm.

3.4.1.3 Combined Prediction Uncertainty

The combined prediction uncertainty is the sum of all model and prediction uncertainties. As both are assumed to be normal distributed, the combined error distribution $pdf(e_1)$ will also be normal distributed. This combined error has a mean as described in Equation 44 and a standard deviation as described in Equation 45.

$$\mu_1^E = 0.4 + 0.0 = 0.4mm$$

Equation 44

$$\sigma_1^E = \sqrt{0.45^2 + 0.05^2} = 0.4528mm$$

Equation 45

3.4.2 Uncertainty Evaluation

The I-beam is now evaluated with respect to uncertainty. The initial design is the design with the optimal part cost subject to noise as shown in Table 12, satisfying the quality requirement in Equation 46. However, this design does not include the effects of uncertainty.

Table 12: Optimal Beam Design under Noise

Variable	Nom.	Value	Response	Nom.	Value
Beam Width	x_1	34.8	Deflection	y_1	2.893
Modulus	x_2	182,150	Marginal Part Cost	C^M	1.64

$$USL^U = USL^N - \alpha \cdot \sigma_1^N = 3 - 3 \cdot 0.034 = 2.899$$

Equation 46

If the mean prediction error is included, the response distribution has a mean deflection of 2.57mm, and a standard deviation of 0.453mm. Due to the large mean prediction error, there is an 81% chance that the prediction uncertainty will violate the quality requirement as visualized in Figure 15.

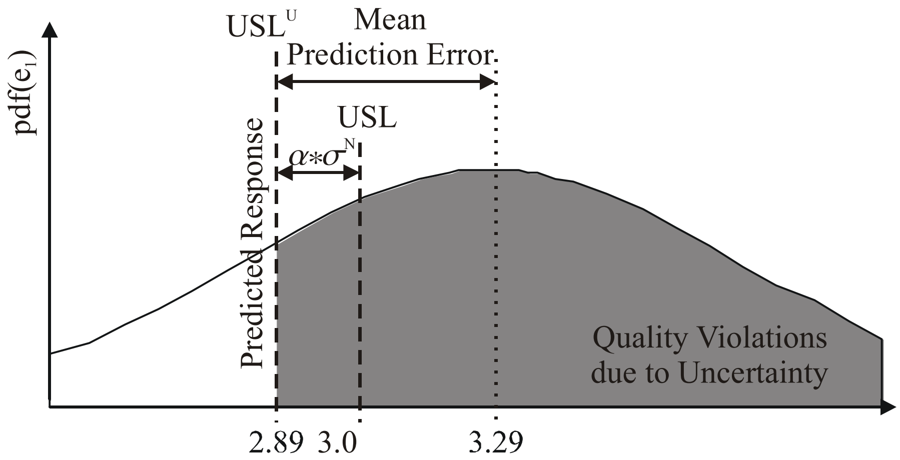


Figure 15: I-Beam Prediction Error

It is possible to improve the beam design down to a zero possibility of violating the quality requirement due to uncertainty. However, this design would cost significantly more than the original design shown in Chapter 2. In this case, it may be more economic to change the design once the prediction error is known. This will be described in the flexible design methodology in the next chapter.

3.5 Conclusions

The uncertainty of a prediction model may have a significant impact on the quality of the product. However, current methods for handling uncertainty are insufficient in generating a trade-off between the cost of the part and the cost of possible changes. The next chapter will discuss a method to select an economic design robust against noise, yet allow a flexible design change towards a design robust against noise and uncertainty in case the uncertainty causes an undesirable design. This avoids the additional cost required to reduce the sensitivity of the design to prediction uncertainty. The methodology aims to reduce the overall cost of the design including the cost of possible design changes.

CHAPTER 4

FLEXIBLE DESIGN METHODOLOGY

4.1 Introduction

The goal of robust design methodologies is to reduce the sensitivity of the designs to performance variation. Although robust design methodologies consider noise, they usually do not account for uncertainties and inaccuracies in the predictions of the design performance. As such, the finely tuned robust design might violate specifications because the underlying predictions lack the necessary accuracy. It is possible to include the uncertainty variation into the robust design evaluation to reduce the sensitivity to noise and uncertainty, yet this could increase the cost of the product while generating little value for the customer. The described methodology aims to minimize the expected cost of the design including development uncertainties.

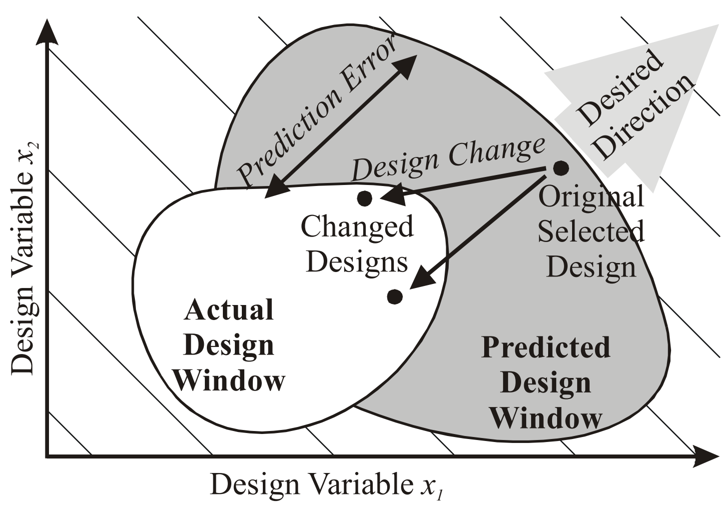


Figure 16: Flexible Design Incentive

Figure 16 shows a predicted feasible design region within which a design is assumed to be feasible. From this predicted design region, a design is selected according to an objective function. The objective function is shown in the graph as the diagonal contour lines. However, the actual and predicted design windows might not coincide due to uncertainties in the development process. If the selected design lies outside of the actual design window, a design change is necessary, even if the model predicted this design to be optimal. This required design change will result in unforeseen development costs, and may also alter the performance and cost of the product.

Within this methodology, the possible expected outcomes are analyzed for a selected initial design. For any given defect, there exists the possibility of changing numerous design parameters in order to resolve the defect. The flexible design methodology investigates all possible design changes for all possible expected outcomes to assist the designer to reduce the overall expected cost of the design including possible future design changes.

Figure 17 gives an overview of the flexible design methodology. The method starts by selecting an initial investigated design, for which prediction models are known. The possible expected outcomes for this design are determined and their likelihood of occurrence evaluated. Based on these expected outcomes, the possible design changes are investigated, and their ability to resolve the expected outcome analyzed. A probabilistic evaluation determines the likelihood of a design change occurring. The overall expected cost of the design is evaluated. If an ideal trade-off between the part cost and the risk of design changes is found, the design will be produced, otherwise the product and process is redesigned and the flexibility is evaluated for the redesigned design.

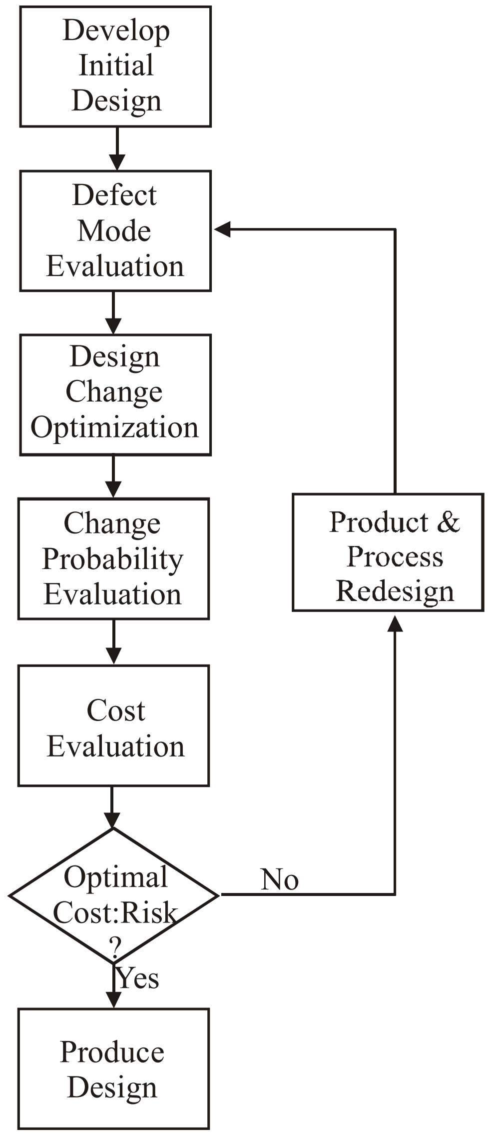


Figure 17: Methodology Outline

The described method builds on other related research. (Thornton 1999b; Thornton 1999c; Thornton et al. 1999) describes the changes in the design variation as the model uncertainties are reduced during the design development, and new information becomes available.(Yoshimura and Nishikawa 1996) considers a flexible design that can adapt to changes on the basis of a trade-off analysis of a Pareto optimal set. (Jordan and Graves 1995) analyzes manufacturing flexibility to adapt the manufacturing capacity to changing market demand. This chapter is also based, in part, on previous work about the flexible design methodology (Roser and Kazmer 1999)

4.2 Expected Outcomes

Each design has a possibility of being defective. As described in Chapter 3, a design is considered infeasible if one or more response y_j is outside of the specification limit LSL^U_j and USL^U_j of the uncertainty. This condition represents a high likelihood that the design will violate the noise specification limits LSL^N_j and USL^N_j and may subsequently require a design change. Therefore, it is necessary to list all possible expected outcomes of the design and evaluate the probability of each expected outcome occurring.

4.2.1 Determination of All Possible Expected Outcomes

A design response y_j with a two-sided specification limit LSL^U_j and USL^U_j could have three different expected outcomes under uncertainty. It is possible that the response y_j violates the lower specification limit for uncertainty LSL^U_j. Second, it is also possible that the response y_j violates the upper specification limit for uncertainty USL^U_j. Third, it is possible that the response y_j violates none of the specification limits LSL^U_j and USL^U_j. This yields for n responses 3^n possible expected outcomes.

These expected outcomes are summarized in a matrix *M*. This matrix consists of one column for each of the *n* specified response y_j and one row for each of the 3^n possible expected outcomes. A single expected outcome is represented by one row M_k of the matrix *M*. The matrix element $M_{k,j}$ contains a -1 one if the expected outcome M_k violates the upper limit USL^U_j for a response y_j, representing the need to reduce the response value in order to obtain a feasible design. The matrix element $M_{k,j}$ contains a +1 if the expected outcome M_k violates the lower limit LSL^U_j for a response y_j, representing the need to increase the response value in order to obtain a feasible design, and the matrix element $M_{k,j}$ contains zero if the specification is not violated.

$$M = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ -1 & -1 \\ -1 & 1 \\ 1 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Equation 47

For two specified design responses y_j there are nine resulting expected outcomes M_k as explicitly shown in Equation 47. The first expected outcome M_1 represents a design, where the lower and upper specification limits under uncertainty for both responses y_j are satisfied. The second expected outcome M_2 represents a violation of the upper specification limit for the second response y_2, while the first response y_1 is within the given specification limits. The last expected outcome M_9 represents both responses y_j violating the lower specification limits LSL^U_j under uncertainty. The next sections discuss the probability of a certain expected outcome occurring.

4.2.2 Probability of Violating One Specification

As described in Chapter 3, the response y_j including the prediction error e_j has to be within the lower and upper specification limits for the uncertainty LSL^U_j and USL^U_j. This prediction error is known only as a probability distribution $pdf(e_j)$, creating a probability distribution of the response under uncertainty $pdf^U(y_j)$. The probability P^L_j of a response y_j violating the lower specification limit LSL^U_j and the probability P^U_j of violating the upper specification limit USL^U_j as shown in Chapter 3 is visualized in Figure 18. Note that for one-sided specifications the probability of violating the other side is zero, as a nonexistent specification cannot be violated. This can also be represented mathematically by setting the corresponding specification to ±∞ or by the removal of the case from the expected outcome matrix M.

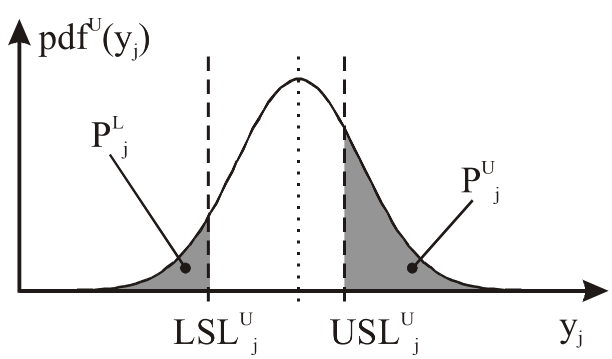


Figure 18: Defect Probability

4.2.3 Probability of a Expected Outcome Occurring

The probability $P^M{}_k$ of a certain expected outcome occurring can be evaluated from the above probabilities $P^L{}_j$ and $P^U{}_j$ of violating the lower or upper specification $LSL^U{}_j$ and $USL^U{}_j$ for a given design response. This probability is the combination of the individual probabilities of specification satisfaction or violation depending on the expected outcome as shown in Equation 48. Due to the feasibility requirement for the initial design, the probabilities $P^L{}_j$ and $P^U{}_j$ of violating the lower and upper specification limits $LSL^U{}_j$ and $USL^U{}_j$ are exclusive. Therefore, the probability of a response y_j being within the specified limits $LSL^U{}_j$ and $USL^U{}_j$ is the difference between certainty and the sum of the violation probabilities $P^L{}_j$ and $P^U{}_j$.

$$P_k^M = \left(\prod_{j=1}^{n} \begin{cases} P_j^U & if\ M_{k,j} = -1 \\ P_j^L & if\ M_{k,j} = 1 \\ 1 - \left[P_j^U + P_j^L \right] & if\ M_{k,j} = 0 \end{cases} \right) + cov \quad \forall k$$

Equation 48

Figure 19 visualizes the Bayesian network of the different expected outcomes for a selected initial design. There are 3^n different expected outcomes M_k, each having a probability of occurring of P_k. The next section will describe the possible design changes to resolve the different expected outcomes M_k.

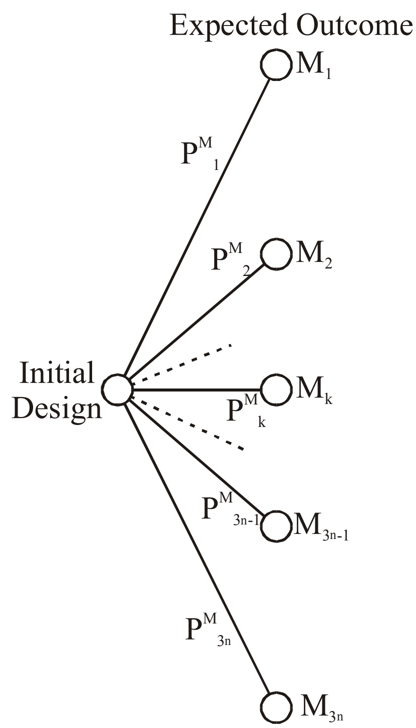


Figure 19: Expected Outcomes

4.3 Deterministic Design Change Analysis

The previous section determined the probability $P^M{}_k$ of different expected outcomes M_k occurring. This section will now investigate different design change options for every expected outcome M_k. For every possible design change $S_{k,l}$ for every expected outcome M_k, the possible design improvement is determined, and the probability of satisfying all specifications $P^D{}_{k,l}$ is evaluated. Based on $P^D{}_{k,l}$ the probability of a change being utilized $P^C{}_{k,l}$ is evaluated.

Please note that for this deterministic design change analysis, it is assumed that the outcome of a design change is known. Hence, there is no uncertainty if a design change will resolve a defect. This is an optimistic approach. In reality, the outcome of a design change is also subject to uncertainty. Therefore, an improved design change analysis under uncertainty is provided in Chapter 5. However, this improved analysis requires knowledge regarding the uncertainty in a design change. Since this information will not always be available, two design change analysis methods are described in this dissertation. The deterministic design change analysis method is described in this chapter, and the design change analysis under uncertainty is described in Chapter 5. Depending on the availability of the information of the design change uncertainty, either method may be utilized.

4.3.1 Design Change Options

In order to determine the options to resolve a expected outcome M_k, possible combinations of changes in the design variables are analyzed with respect to cost and impact on the design performance. If there exist m investigated design variables, there will be 2^m possible design change combinations S_l, representing 2^m subsets of the m dimensional design space. As each additional investigated variable doubles the number of investigated design changes, the computation time will increase exponentially. Therefore, only significant variables should be included in the methodology. An example set S of possible design changes S_l for three design variables is shown below in Equation 49.

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Equation 49

Figure 20 represents the design space and subspaces for the above set of design changes *S*. The first change option S_1 represents changing no variables of *X* and keeping the initial design. This is represented by the zero dimensional sub space in Figure 20. The second design change option, S_2 represents only changing variable x_3, while keeping the variables x_1 and x_2 at the initial value. This is represented as the vertical one dimensional sub space in Figure 20.

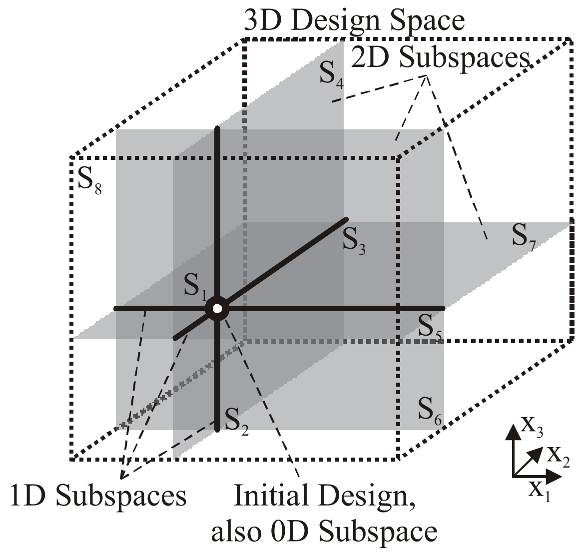


Figure 20: Design Space and Sub Spaces

4.3.2 Response Change

Figure 21 shows an overview of the possible design change options. These design change options are identical for all expected outcomes. However, the actual design changes will differ from each other. The following sections describe the criteria used for the design change evaluation. It is important to note that the prediction error is not known exactly. Hence, it is not known what change is necessary. Rather, the described method estimates the likelihood of changes based on the information available at the design development phase.

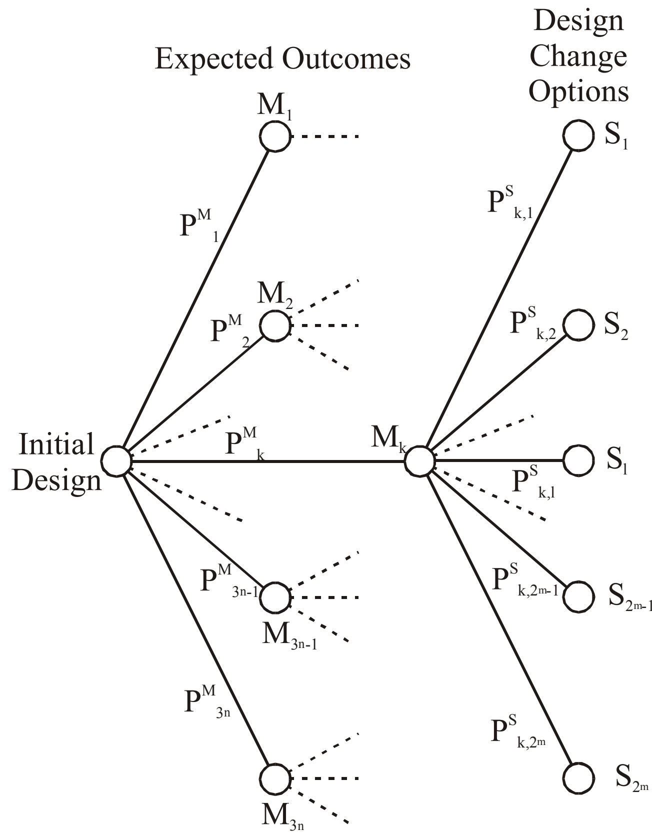


Figure 21: Design Change Options

The goal of a possible design change S_l is to move the responses y_j violating the specification limits LSL^U and USL^U in a direction away from the violated specification limit, while having the non-violated responses within the specification limits LSL^U and USL^U. This change will move the actual response away from the violated specification limit. It is important to point out, that the design change optimization still uses the original prediction model. If the prediction model shows inverse trends compared to the actual design space, then the validity of the design change degrades. The methodology is able to handle offset errors, but not topological errors in the prediction model.

4.3.3 Conditional Probability of Defect Satisfaction

The design change aims to move the responses away from the violated specifications while keeping the non-violated responses within the specifications. The final design change cannot be determined during the design development stage since the exact prediction error is not known. However, depending on the expected outcome, assumptions can be made regarding the uncertainty distribution. This is visualized in Figure 22, where a prediction uncertainty distribution may violate the upper or lower specification limit. Depending on the type of specification violation, certain prediction errors can be excluded, and the uncertainty distribution is reduced. For example, if the expected outcome assumes a violation of the lower specification limit, the possible prediction errors are reduced to the left tail of the distribution as shown in graph (a) of Figure 22. If the expected outcome assumes a violation of the upper specification limit, the uncertainty can be expressed as the right tail in graph (c). If neither limit is violated, the uncertainty distribution is the section between the limits as shown in graph (b). Furthermore, it has to be ensured that the area underneath each individual probability distribution integrates to one. It is important to point out that the conditional distributions assume that a fixed prediction error measured at one point in the design space to be valid for the complete design space. However, the prediction error is likely not to be constant for the complete design space but may vary throughout the design space. After the creation of the physical design, it is recommended to improve the prediction rather than to assume a fixed error.

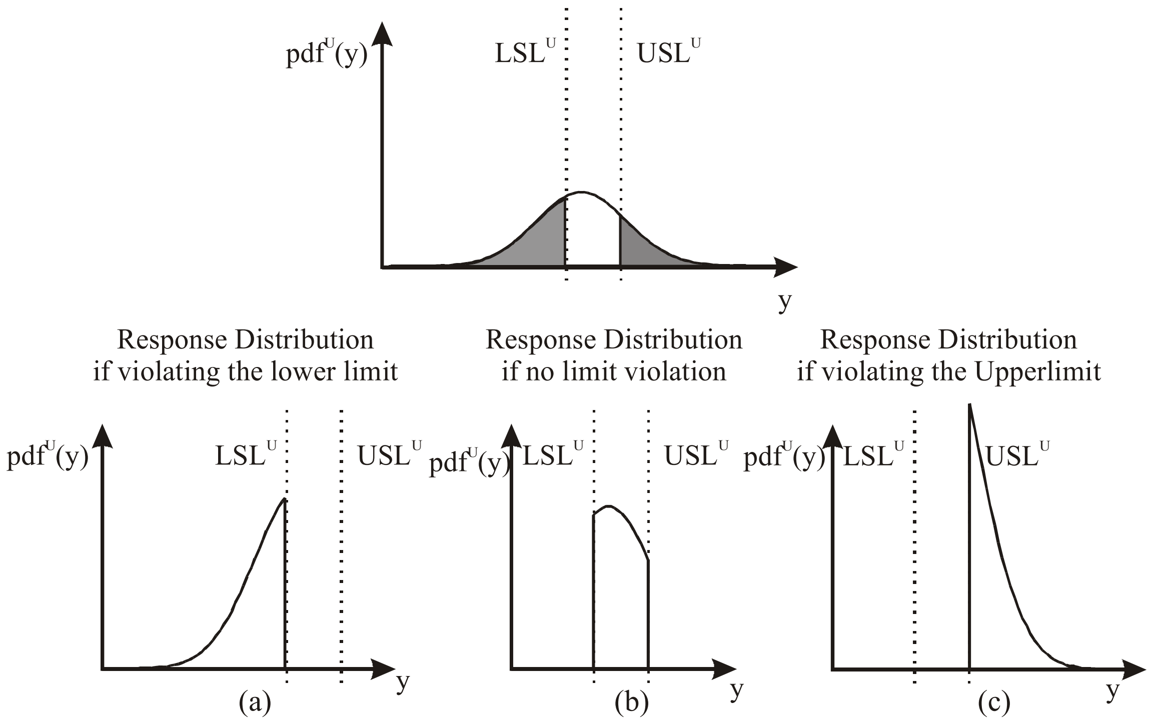


Figure 22: Prediction Error Distribution for Satisfaction or Violation

Due to the design dependencies, moving one response in one direction might move another response in an undesired direction, and potentially violate another specification. Therefore, each of these resulting response distributions might violate either the lower specification limit or the upper specification limit under uncertainty after the design change. The probability of violating no specification limit can be described as an integral of the response distribution between the specification limits. However, it is also possible to describe it in terms of the original design violating the lower and upper specification limits $P^{L}_{,j}$ and P^{U}_{j}, and the current changed design violating the lower and upper specification limits $P^{L}_{k,l,j}$ and $P^{U}_{\mathrm{k,l,j}}$, where the indices k, l and j stand for the expected outcome M_k, the design change S_l and the design response y_j. This is shown in Equation 50, Equation 51, and Equation 52 where the probability $P^{D}_{k,l,j}$ of satisfying a specification after a design change is evaluated for the condition of previously violating the upper limit, the lower limit, and no limit under uncertainty. If the probability $P^M{}_k$ of the expected outcome occurring is zero, then this defect never occurs and all subsequent probabilities will be zero.

$$P^D_{k,l,j} = 1 - \left[\frac{Min\left[P^U_j\,,\, P^U_{k,l,j}\right]}{P^U_j} + \frac{Max\left[\left(P^L_{k,l,j} + P^U_j - 1\right), 0\right]}{P^U_j} \right] \quad if\ M_k = -1, P^M_k > 0, k > 1, l > 1$$

Equation 50

$$P^D_{k,l,j} = 1 - \left[\frac{Min\left[P^L_j\,,\, P^L_{k,l,j}\right]}{P^L_j} + \frac{Max\left[\left(P^U_{k,l,j} + P^L_j - 1\right), 0\right]}{P^L_j} \right] \quad if\ M_k = -1, P^M_k > 0, k > 1, l > 1$$

Equation 51

$$P^D_{k,l,j} = 1 - \left[\frac{Max\left[\left(Min\left[P^U_{k,l,j}\,,\, 1 - P^L_j\right] - P^U_j\right), 0\right] + Max\left[\left(Min\left[P^L_{k,l,j}\,,\, 1 - P^U_j\right] - P^L_j\right), 0\right]}{1 - \left(P^L_j + P^U_j\right)} \right]$$
$$if\ M_k = 0, P^M_k > 0, k > 1, l > 1$$

Equation 52

These equations consist out of two parts. The likelihood of the conditional uncertainty distribution violating the lower specification limit and the conditional probability of violating the upper specification limit under uncertainty is determined. The probability of satisfying the specifications is the remainder to certainty. In Equation 50, for example, the probability of satisfying all specification limits is one minus the probability of violating any specification limits under uncertainty. This probability consists of the probability of violating the upper specification limit and the probability of violating the lower specification limit respectively based on the conditional uncertainty distribution.

Based on the above probability of resolving one specification violation $P^D{}_{k,l,j}$ the joint probability of resolving all specification violations $P^D{}_{k,l}$ for a given expected outcome M_k and design change S_l has to be calculated. This is shown in Equation 53.

$$P_{k,l}^{D} = \prod_{j=1}^{n} P_{k,l,j}^{D} + cov$$

Equation 53

The design change aims to improve the probability $P^D{}_{k,l}$ of satisfying all specifications given the current expected outcome M_k by means of a given design change S_l. However, there are certain exceptions, where the probability of resolving all defects $P^D{}_{k,l}$ is known beforehand. Hence, it is not necessary to evaluate the design changes for these cases.

4.3.4 Known Probabilities of Specification Satisfaction

The following method will evaluate the possible design changes including the cost of the design change and the probability of the design change occurring. However, out of the 2^m design changes for the entire 3^n expected outcomes some cases can be determined beforehand using common sense. These exceptions are described below.

If a expected outcome M_k does not exist, then the probability of satisfying all specifications after a design change $P^D{}_{k,l}$ for all possible design changes S_l for this expected outcome M_k has no influence on the design evaluation. Subsequently the probability of satisfaction $P^D{}_{k,l}$ is set to zero for all design changes S_l for expected outcomes M_k with zero probability of occurrence $P^M{}_k$ as shown in Equation 54. The computation time can be reduced by not analyzing the design changes for a nonexistent expected outcome.

$$P_{k,l}^D = 0 \ if \ P_k^M = 0 \ \forall l$$

Equation 54

Another expected outcome M_k for which the design change is known beforehand is the first expected outcome M_1, representing the case where all design variables Y are within the given uncertainty limits LSL^U and USL^U. If there is no violation of any specification limits under uncertainty, then there exists no need to change the design. Hence, the design change option for the first expected outcome M_1 is not to change the design $S_{1,1}$. Hence, the probability $P^D{}_{1,1}$ of satisfying the (non-existent) defect is 1, and the probability of satisfying the defect $P^D{}_{1,l}$ by means of any other design change option $S_{1,l}$ is zero.

$$P_{1,1}^D = 1$$
$$P_{1,l}^D = 0 \ \forall (l > 1)$$

Equation 55

Finally, if a design is defective, but no change is performed, then a design failure occurs. This probability of failing to create a feasible design will be discussed in more detail in section 4.3.8.

4.3.5 Design Change Optimization

The design change aims to improve the probability of satisfying all defects $P^D{}_{k,l}$ for a given expected outcome M_k by means of a given design change S_l. This optimization has to be performed for every design change option S_l for every expected outcome M_k excluding the exceptions described in section 4.3.4.

$$
\begin{aligned}
Max \quad P_{k,l}^{D} \quad s.t. \quad & LSL_j^U \le y_j \le USL_j^U \ \ if\ M_{k,j} = 0 \\
& LCL_i \le x_i \le UCL_i \ \ if\ S_{l,i} = 1 \\
& x_{k,l,i} = x_{k,l,1} \ \ if\ S_{l,i} = 0 \\
& k \in \left[1, 3^n\right], l \in \left[1, 2^m\right]
\end{aligned}
$$

Equation 56

4.3.6 Cost of the Design

In order to select between different design changes S_l capable of resolving a given defect M_k, the marginal cost including the amortized cost of the design change. This analysis requires the structuring of the tasks necessary to change a design parameter. This model structure is related to design task modeling, where a development process is divided into sub tasks. (Steward 1981) describes the design structure matrix as an approach to manage complex design systems. This approach is extended for the change cost analysis.

The relation between the design variables x_i and the tasks ξ_q required for changing these design variables are represented in a matrix ξ. This matrix consists of one row for each design variable x_i, and one column for each possible task ξ_q. If a change in a design variable x_i requires the execution of task ξ_q, a one will be inserted in the matrix ξ in row i and column q. Equation 57 shows a matrix ξ representing the relation between three design variables and five tasks.

$$\xi = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Equation 57

A task is required if at least one changed design variable requires the performance of the task. This is done by multiplying the vector of the design change cost with the matrix ξ. The resulting matrix elements have values larger than zero for each task ξ_q required to execute the set of variables S_l. Furthermore, each task ξ_q creates a cost $C^{\xi}_{k,l}$ during execution. The change cost $C^{C}_{k,l}$ (per unit) of the design change evaluates as the sum of the cost of all executed tasks divided by the production volume V:

$$C_{k,l}^{C} = \frac{\sum_{q=1}^{r} \begin{cases} C_r^{\xi} & if \left[S_l \cdot \xi\right]_q > 0 \\ 0 & else \end{cases}}{V}$$

Equation 58

Please note that the above method for determining the cost of a design change is a very general approach, and estimates the change cost merely based on the changed variables. The change cost might differ depending on the change direction or magnitude, for example if a hole diameter in a tool has to be reduced rather than increased. These asymmetric cost relations are not modeled within this system. Improved methodologies for change cost estimations can be developed and used within this methodology. (Martel 1988) for example describes the cost of an engineering change order in printed circuit design as a range from $1,200 to $6,000, with an average of 30 changes per week for an average company. This calculates to about $4.5 million per year for an average company. (Lavoie 1979) also describes non-technical considerations in design changes, as for example the behavior of the market and legal complications. (Lenane 1986) also describes the cost of defects, depending on the effort required to fix the design as for example rework or excessive material handling. (Lundvall) categorizes the cost of defects and describes methods on how to measure and improve quality.

The marginal part cost $C^{M}_{k,l}$ for a given design change S_l and a given failure mode M_k is required as part of the initial functional relations needed to perform the flexible design methodology. Together with the change cost $C^{C}_{k,l}$ it is now possible to evaluate the total cost $C^{T}_{k,l}$ of the design change S_l for a given failure mode M_k as shown in Equation 59.

$$C^{T}_{k,l} = C^{M}_{k,l} + C^{C}_{k,l}$$

Equation 59

4.3.7 Probability of Design Change

After performing the above optimization for all possible design changes S_l and all possible expected outcomes M_k, a total of 3^n times 2^m design changes are evaluated. The probability of satisfying all specifications $P^D_{k,l}$ and the total cost of a changed design $C^T_{k,l}$ can now be used to determine the likelihood of selecting a given design change from the set of possible design changes S for a given expected outcome M_k. With respect to economic considerations, the different design changes S_l for a given expected outcome M_k are sorted by the total cost $C^T_{k,t}$, where index t ranging from 1 to 2^m refers to the previously unsorted index l as shown in Table 13.

Table 13: Sorted Design Changes for Expected Outcome M_k

Design Change	Total Cost	Sorting Criteria	Probability of Satisfaction
$S_{k,1}$	C^T_{k,t_1}	-	P^D_{k,t_1}
$S_{k,2}$	C^T_{k,t_2}	$C^T_{k,t_2} > C^T_{k,t_1}$	P^D_{k,t_2}
…			
$S_{k,t}$	C^T_{k,t_l}	$C^T_{k,t_l} > C^T_{k,t_{l-1}}$	P^D_{k,t_l}
$S_{k,2m}$	$C^T_{k,t_{2m}}$	$C^T_{k,t_{2m}} > C^T_{k,t_{2m-1}}$	$P^D_{k,t_{2m}}$

For a given prediction error E, there might be more than one design change $S_{k,t}$ capable of resolving the defect. With respect to economic considerations, the design change with the least total cost C^T_{k,t_l} would be selected from the list of possible design changes S for a given expected outcome M_k to resolve the defect. Subsequently, a design change would only be selected if the change S_{k,t_l} resolves the defect and all more economic design changes do not resolve the defect. The calculation of the probability P^S_{k,t_l} of a design change occurring is shown in Equation 60.

$$P^S_{k,t_l} = P\left(P^D_{k,t_l} \cap \overline{P^D_{k,t_{l-1}}} \cap \overline{P^D_{k,t_{l-2}}} \cap \cdots \cap \overline{P^D_{k,t_2}} \cap \overline{P^D_{k,t_1}}\right)$$

Equation 60

To calculate the actual value of selecting a design change S_{k,t_l} from the list of design changes S for a given expected outcome M_k, interactions have to be taken under consideration. Independence cannot be assumed. This is shown in Equation 61.

$$P^S_{k,t_l} = \left(P^D_{k,t_l} \cdot \prod_{u=1}^{l-1}\left(1 - P^D_{k,t_l}\right)\right) + cov$$

Equation 61

However, it is possible to simplify Equation 61 using a reasonable dependence assumption. Figure 23 shows one response distribution for an initial design and two possible design changes. It can be seen that design change two will resolve all defects which where also resolved by design change one. Hence, the probability of change two resolving defects not resolved by change one is the difference between the two probabilities as shown in Equation 62.

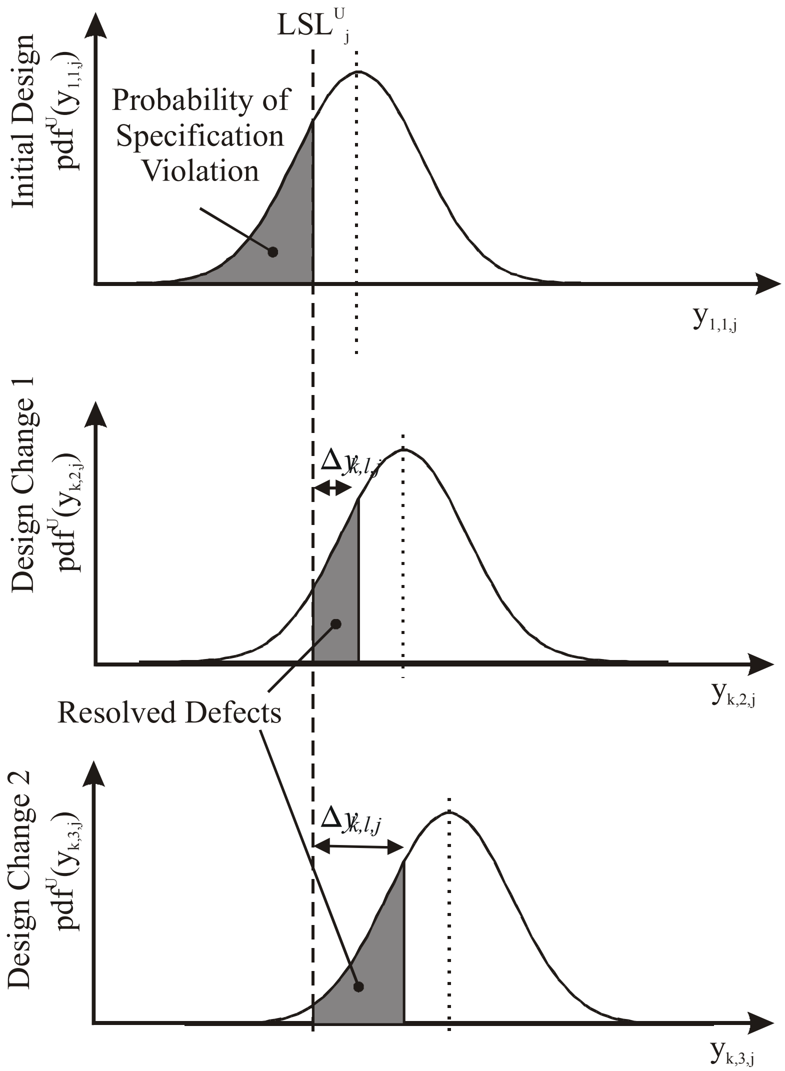


Figure 23: Interaction Assumption

$$P^S_{k,t_l} = \begin{cases} P^D_{k,t_1} & if\ l = 1 \\ P^D_{k,t_l} - max\left(P^D_{k,t_1} \cdots P^D_{k,t_{l-1}}\right) & else \end{cases} \quad \forall k, l$$

Equation 62

The probability of a certain design change occurring $P^C{}_{k,t}$ depends on the probability of selecting this design change $P^S{}_{k,t}$ from the list of possible design changes S

for a given expected outcome M_k and the joint probability of the expected outcome occurring $P^M{}_k$:

$$P_{k,t}^C = P_k^M \cdot P_{k,t}^S \qquad \forall k, t$$

Equation 63

4.3.8 Probability of Design Failure

For every expected outcome M_k there exists a probability of not being able to resolve the defect. This probability of not being able to satisfy the requirements is represented as the probability of not changing the design $P^S{}_{k,1}$ despite a defect for the given expected outcome M_k. Due to the significance of this term on the success of the design this is nominated as the probability of failure $P^F{}_k$ for each expected outcome M_k as shown in Equation 64.

$$P_k^F = P_{k,1}^S = 1 - \sum_{l=2}^{2^m} P_{k,t_l}^S \qquad \forall k > 1$$

Equation 64

As a single expected outcome might not be resolved, the probability of design failure P^F for all expected outcomes M can be determined as shown in Equation 65. The probabilities of a certain design change are summed over all expected outcomes and design changes, excluding the first expected outcome and the first design change.

$$P^F = \sum_{k=2}^{3^n} P_k^M \cdot P_k^F$$

Equation 65

The probability of design failure represents the likelihood of the initial design not being able to satisfy the quality requirements including the possibility of a design change, based on the available knowledge during the design development stage. Therefore, a design failure as described in the above context does not necessarily mean the inability to create a design that satisfies the quality requirements. Rather, it represents the inability to satisfy the quality requirements using the given design system and variation information.

As additional knowledge is gained during the development of the design, better information regarding the prediction error and noise distribution becomes available, and a better prediction about the likelihood of satisfying the quality requirements can be made. The information regarding the probability of failure is restricted to the modeled design space. It may be possible to adjust design parameters not modeled within the methodology, or to extend the range of the design variables. In addition, a change in the design concept might be able to resolve the defect despite large prediction errors. Finally, a relaxation of the quality requirements may resolve the defects. Therefore it is important to note that the probability of design failure merely represents the inability of the given design system using the current uncertainty information to satisfy the specified customer requirements.

As the expected cost C^E is measured monetarily, a cost has to be related to the design failure C^F. This cost occurs if a defect design remains unchanged as shown in

Equation 66. The cost of failure represents the additional effort required to expand the design model, investigate a different design concept, or relax the quality requirements. It is also possible to cancel the design project, in which case loss of market share, sunk development cost, penalty fees, and loss of reputation might occur. The estimated failure cost is outside the scope of this research. Thus, it is necessary to gather estimates from human expertise.

$$C_{k,1}^{T} = C^{F} \quad \forall k > 1$$

Equation 66

4.4 Evaluation

The results of the flexible design analysis can now be used to draw conclusions regarding the flexibility of the design. The results can be used to estimate the expected cost of the design C^E and the likelihood of changing a design variable x_i or violating a design response y_j.

4.4.1 Expected Cost

The expected cost C^E is the average cost of the design including all possible changes and failures for all expected outcomes. It is important, however, to note that this cost is a probabilistic average of different expected outcomes and design changes. As only one design is created, there will be only one certain expected outcome with one selected design change. This case is not known until the design is created and the actual prediction errors E occur. Therefore, the design might cost more or less than expected, yet the average cost will be the expected cost C^E. The expected cost can be calculated for each expected outcome $C^E{}_k$, and then for the overall design. Equation 67 shows the relation between the expected cost $C^E{}_k$ and the cost and probabilities of the individual design changes for a given expected outcome M_k. The overall expected cost C^E for the initial design is the sum of all costs $C^E{}_k$ as shown in Equation 68.

$$C_k^E = \sum_{l=1}^{2^m} C_{k,t_l}^T \cdot P_{k,t_l}^C$$

Equation 67

$$C^E = \sum_{k=1}^{3^n} C_k^E$$

Equation 68

The expected cost is risk neutral, as the flexible design methodology tries to balance the cost of the design with the risk of a design change. However, depending on the design project, the company and the market environment, some design projects use an optimistic, i.e. risk prone approach, where a low part cost is more important than a possible design change, whereas other design projects might be pessimistic, i.e. risk adverse, where the cost of the design is less important than the possible design changes. (Thornton 1999a) presents an excellent overview of optimistic and pessimistic design under uncertainty and describes which approach might be suitable depending on the design environment. This depends in part on the capability of the manufacturing process. The usage of the process capability has been investigated by (Tata and Thornton 1999).

4.4.2 Probability of Design Variable Change

Using the estimated probabilities, it is possible to determine the likelihood of changing a certain design variable x_i. It is also possible to distinguish between the likelihood of an increase and a decrease of the value of the design variable. Therefore, the probability $P^X{}_i$ of changing the design variable x_i can be evaluated. This evaluation is based on the probability of a certain design change $P^C{}_{k,t}$, which is known for all expected outcomes M_k and design changes S_l. Hence, it is possible to evaluate the probability of a variable changing $P^X{}_i$ as the sum of the probabilities $P^C{}_{k,t}$ of design changes $S_{k,t}$ including this variable x_i as shown in Equation 69, where $S_{k,t_l,i}$ equals one if x_i is changed and zero otherwise.

$$P_i^X = \sum_{k=1}^{3^n} \sum_{l=1}^{2^m} P_{k,t_l}^C \cdot S_{k,t_l,i}$$

Equation 69

4.4.3 Probability of Design Response Defect

Other valuable information about the design flexibility includes the probability of a defect $P^Y{}_j$ due to a certain response violation y_j. This probability $P^Y{}_j$ is simply the sum of the probabilities $P^L{}_j$ and $P^U{}_j$ of violating the lower and the upper specification limit LSL^U and USL^U under uncertainty described in section 4.2.2 and shown in Equation 70.

$$P_j^Y = P_j^L + P_j^U$$

Equation 70

4.5 Design Improvement

The flexible design methodology generates an objective including the marginal cost of the design and the cost due to possible design changes. Using the described methodology, it is now possible for the designer to improve the expected cost by increasing the flexibility of the design. Improved designs reduce the probability of costly design changes. A search algorithm can be utilized to determine the minimum expected cost, representing the design with the smallest overall expenses with respect to possible design changes. While there is no exact method developed to recommend an improved design, it is recommended to reduce the probability of changing design variables with a large change cost. This redesign is likely to increase the marginal cost, yet may reduce the overall expected cost. Secondly, it may be possible to reduce the increased marginal costs by adjusting design variables with a small change cost. Overall, this approach increases the design flexibility, where design variables with a large change cost are likely to remain unchanged, yet variables with a small change cost add degrees of freedom to the design to adjust for uncertainty and minimize cost.

4.6 Example: I-Beam

The flexible design methodology will be demonstrated on the I-beam example. The optimal design from chapter 2 will be analyzed using the flexible design methodology. The objective is to reduce the expected cost by reducing the sensitivity to uncertainty, i.e. trying to achieve a trade-off between the cost of the design and the likelihood and cost of the changes. However, before the beam can be analyzed using the flexible design methodology, some additional information is required.

4.6.1 Assumptions

The flexible design method is demonstrated using the response surface models introduced in Chapter 2. These response surface models are assumed to have prediction accuracy as described in Chapter 3, with a mean error of 0.4mm and a standard deviation of the error distribution of 0.45mm. The flexible design methodology also requires the cost of changing a design variable. Fixed change costs are assumed within this example. A change in the beam height x_1 would require retooling of the extrusion tool. This is assumed to cost \$15,000, giving a change cost of \$0.30 per part for 50,000 parts. A change in the material x_2 is significantly less expensive, with a total cost of \$50, giving a change cost of \$0.001 per part. The cost of changing both the beam height and the modulus is the sum of the two individual changes, equal to \$0.301 per part. The cost of a design failure, i.e. the inability to satisfy the design using the given design relations and therefore requiring a redesign, is assumed to be \$250,000, giving a failure cost C^F of \$5 per part.

4.6.2 Flexible Design Evaluation

The flexible design methodology will be demonstrated using the initial design shown in Table 14. The values of the design responses are shown in Table 15. As derived in Chapter 2, this design has the least marginal cost that meets the quality requirements.

Table 14: Initial Design Variables

Design Variable	Nom.	Value
Beam Height	x_1	34.8mm
Modulus	x_2	182,200 N/mm^2

Table 15: Initial Design Responses

Design Response	Nom.	Value
Deflection Mean	y_1	2.89mm
Marginal Cost	C^M	$1.64

4.6.2.1 Expected Outcomes

The I-beam example has only one response with an upper specification limit. Therefore, there exist only two possible expected outcomes. Either the design satisfies all quality requirements, or the design violates the upper specification limit of the deflection. These two expected outcomes are visualized in Figure 24.

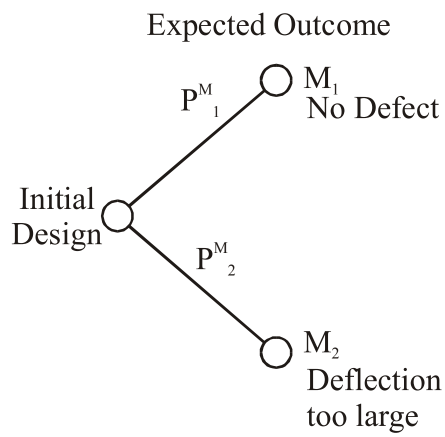


Figure 24: I-Beam Expected Outcomes

The upper specification limit USL^U under uncertainty is three standard deviations away from the specification limit under noise USL^N. For the initial design, the revised specification limit USL^U is 2.899mm. Combined with the uncertainty distribution, this gives the probability of 80.8% of violating the upper specification limit under uncertainty. Therefore, the probability $P^M{}_2$ of the second expected outcome M_2 occurring is 80.8%. Subsequently, the probability $P^M{}_1$ of the first expected outcome M_1, i.e. no defect, occurring is 19.2%.

4.6.2.2 Design Changes

There are four distinct design changes possible for the I-beam example. These change options are listed in Table 16. With two expected outcomes, this would generate eight possible combinations of defects and changes. The first expected outcome represents the situation where no specification is violated. Therefore, no change is necessary, and the design will not be changed. For the second expected outcome, where the specification limit is violated, the design has to be changed. The option not to change the design is only valid if no possible change would resolve the defect. A graphical overview of the possible design changes for the possible expected outcomes is shown in Figure 25. These possible expected outcomes and design changes are also indexed in a table as shown in Table 17. This table form is next utilized to demonstrate the flexible design methodology.

Table 16: I-Beam Design Changes

Change	Index	Changed Variables
No Change	S_1	
Height	S_2	x_1
Modulus	S_3	x_2
Height and Modulus	S_4	x_1, x_2

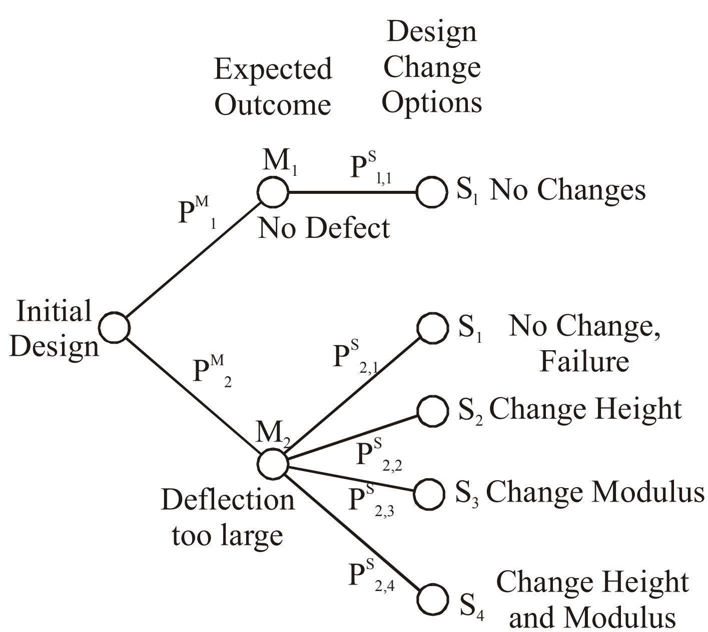


Figure 25: I-Beam Design Change Options

Table 17: I-Beam Design Change Options

Index	Defect	Change	Comment
1	M_1	S_1	No Defect, Unchanged Design
2	M_2	S_1	Deflection Defect, No Change: Design failure
3	M_2	S_2	Deflection Defect, Change Height
4	M_2	S_3	Deflection Defect, Change Modulus
5	M_2	S_4	Deflection Defect, Change Height and Modulus

4.6.2.3 Design Change Optimization

Each design change is optimized to maximize the probability of satisfying all specifications given the current expected outcome as described in 4.3.3. For five different combinations of expected outcomes and design changes, the set of optimizations yield five designs. These designs are listed in Table 18.

Table 18: I-Beam Design Change Values

Index	Defect	Change	Height	Modulus	Total Cost
1	M_1	S_1	34.8	182,200	1.64
2	M_2	S_1	n/a	n/a	5.00
3	M_2	S_2	54.6	182,200	2.32
4	M_2	S_3	34.8	185,000	1.65
5	M_2	S_4	54.6	185,000	2.33

The probability of selecting a certain design change from the possible design changes for a given expected outcome depends on the probability of resolving the defect and the cost of the design. Subsequently, the probability of this change occurring can be calculated with respect to the probability of the defect occurring. An overview is given in Table 19. The rows in this table refer to the same rows in Table 18. The order in which the rows are discussed is selected to improve the understanding of the theory. Note, that the probability of the expected outcome is identical for case 2, 3, 4, and 5, as this is the same expected outcome of violating the upper specification limit for the deflection under uncertainty.

Table 19: I-Beam Design Change Probabilities

Index	Probability of Expected Outcome	Total Cost	Probability of Satisfaction	Probability of Occurrence	Probability of Change
1	19.2	1.64	100.0	100.0	19.2
2	80.8	5.00	n/a	0.00	0.00
3	80.8	2.32	100.0	94.9	76.7
4	80.8	1.65	5.1	5.1	4.1
5	80.8	2.33	100.0	0.00	0.0

The first row represents the option not to change if there is no defect. As this is the only option, it has a likelihood of 100% of being selected from the possible options. As the expected outcome has a probability of 19.2% of occurring, there will be a total probability of 100% times 19.2% equals 19.2% of this combination of expected outcome and design change occurring.

The fourth row is the most economic possible design change for the given expected outcome. As economic design changes are preferred, it is preferable to resolve the defect using this design change if possible. As this change resolves only 5.1% of the defects, it is selected only in 5.1 % of the cases. As the expected outcome occurs with 80.8% likelihood, there is only a 4.1% chance of this change and expected outcome occurring.

The third row is the second most economic design change. This change is used only if the fourth design does not resolve the defect. The probability of occurrence is the probability of resolving the defect if the fourth case does not resolve the defect. The third case therefore has a probability of being selected from the possible changes for a given expected outcome of 94.9%, and subsequently a probability of 76.7% of occurring.

The fifth row is only utilized if all other changes do not resolve the problem. As this case of changing both design variables does not improve the probability of resolving the defect, it will not be selected.

Finally, the second row represents the likelihood of design failure, where the design remains unchanged despite a defect. This case only occurs if all other design changes for this expected outcome fail to resolve the defect. Therefore, the probability of this design change for this expected outcome is the difference between and 100% the sum of all other probabilities of occurrence for the given expected outcome. In this case, there is a probability of 0.00040% of not being able to resolve the defect if a defect occurs. Together with the probability of the defect occurring this gives an overall probability of 0.00032% of design failure, which for all practical matter reduces the probability of failure to approximately zero. It should be noted, however, that even small failure probabilities can have significant impact on the design evaluation when failure costs are very high.

4.6.2.4 Expected Cost

The expected cost, C^E, is shown in Table 20. The I-beam example has an expected cost of $2.17 for the initial design as described above. An overview of the flexible design analysis is shown in Table 21. The results indicate that there is only a 19.2% likelihood of the design satisfying the quality requirement. There is an 80.8% change that the design violates the quality requirement. Most importantly, there is a zero probability of encountering a design flaw, which cannot be compensated.

Table 20: I-Beam Expected Cost

Index	Total Cost	Probability of Change	Product of Cost & Probability
1	1.64	19.2	0.31
2	5.00	0.00	0.00
3	2.32	76.7	1.78
4	1.65	4.1	0.07
5	2.33	0.0	0.00
		Expected Cost	2.17

Table 21: Initial I-Beam Summary

Expected Cost	\$2.07
Initial Height	34.8mm
Initial Modulus	182,200 N/mm^2
Probability of no change	19.2%
Probability of any change	80.8%
Probability of failure	0%
Probability of changing height	76.7%
Probability of changing modulus	4.1%

Looking more closely at the design changes, it can be seen that there is an 76.7% chance that it is necessary to change the beam height. This change requires costly retooling of the extrusion die. The more economic change of the modulus happens only in 4.1% of the cases. To reduce the expected cost, it would be advisable to increase the beam height to reduce the likelihood of costly changes. An alternative design with a smaller expected cost can be found. To reduce the likelihood of changing the beam height, the beam height has been increased from 34.8mm to 40.9mm. This reduces the likelihood of a design change in the beam height, which unfortunately also increases the marginal part cost. However, for the increased wall thickness, it is possible to reduce the marginal part cost by reducing the modulus, increasing the design flexibility.

Table 22: Improved I-Beam Summary

Expected Cost	$1.74
Initial Height	40.9mm
Initial Modulus	149,000 N/mm^2
Probability of no change	70.2%
Probability of any change	29.8%
Probability of failure	0%
Probability of changing height	2.1%
Probability of changing modulus	27.7%

Table 22 shows an overview of the flexible design analysis of the improved design. The likelihood of a defect has been reduced from 80.8% of the previous design in Table 21 to 29.8%. However, 93% of these defects can be resolved by means of a fast and cost efficient change of the modulus. Overall, there is a 70% chance of not changing the design at all, and in almost all other cases, it is possible to adjust for the prediction error by changing the modulus. There is only a 2.8% change that a costly change in the beam height is required, and virtually no chance of a design failure requiring a redesign.

4.6.3 Conclusions

The improved design was able to reduce the expected cost from $2.17 to $1.74, a cost reduction by $0.43, or about 20%. For the estimated 50,000 parts, this would have reduced the total production cost from $108,500 by $21,500 down to $87,00 by reducing expensive design changes and allowing flexibility with fast and efficient design changes.

This would have not only saved money, but also time due to delays in design changes. Figure 26 and Figure 27 compare the design options for the two different initial designs, the first using the design with the least marginal part cost and the second using the design with the least expected cost. The thickness of the connecting lines represents the likelihood of the expected outcome and design change occurring. These trees are mutually exclusive and completely exhaustive, all possible options are shown. The cost of the design options per part is also shown. For the first design in Figure 26, there is a very high likelihood of an expensive change of the beam height. Few designs are feasible without changes, and very few defects can be resolved by changing only the modulus. The second, improved design, exhibits a likelihood of the initial design being feasible. Moreover, a defect can most likely be resolved by means of changing the modulus.

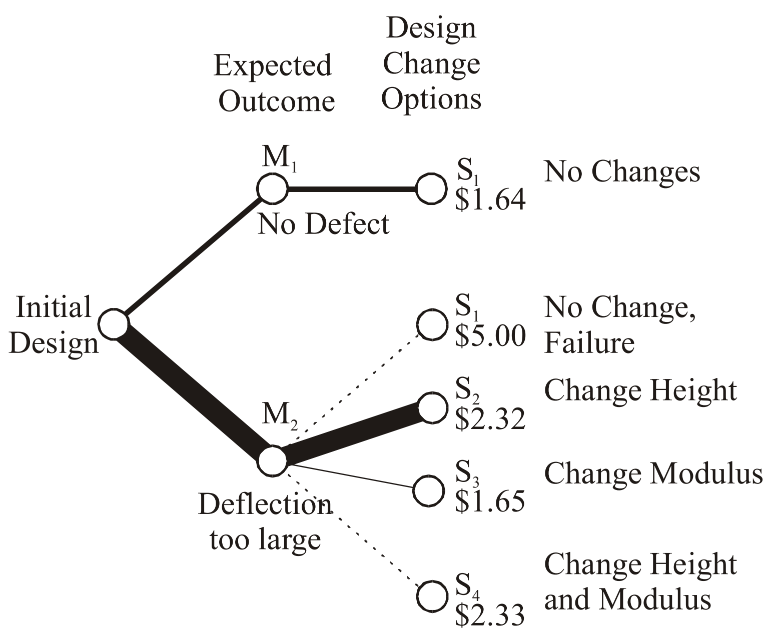


Figure 26: Design Options for Design with Optimal Marginal Cost

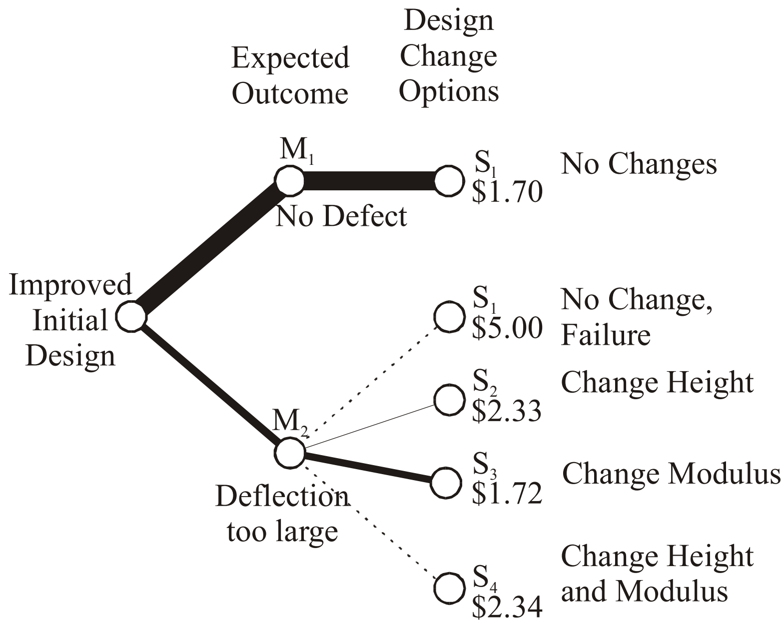


Figure 27: Design Options for Design with Optimal Expected Cost

4.7 Summary

The flexible design methodology aims to reduce the overall cost by reducing the cost of design changes due to prediction errors. The goal is not to improve the robustness against uncertainty, but rather to reduce the negative impact of uncertainty on the cost of the design. The method evaluates the possible expected outcomes due to uncertainty and analyzes the possible design changes. This analysis enables the design team to modify the design in order to improve the flexibility of the design, resolving defects using economic design changes instead of costly and delaying design changes. This approach will be used in Chapter 6 to also determine the value of information for different design models and to investigate the relation between the cost and the use of prediction models. The following chapter presents a method to analyze the design change under uncertainty if the information regarding the design change uncertainty is available.

CHAPTER 5

DESIGN CHANGE ANALYSIS UNDER UNCERTAINTY

5.1 Introduction

The deterministic design change analysis in Chapter 4 evaluated the possible design changes and employed the strategy of selecting the least expensive design change able to resolve the defect. However, in reality it cannot always be guaranteed that a design change will actually resolve the defect, but there exists the possibility that a design change fails to resolve a given defect. This situation is not considered in Chapter 4, where the exact outcome of a design change is assumed to be known. As the design model has uncertainties, a prediction of the design responses will be imprecise. Creating a design will generate accurate information about one point in the design space, yet if the design has to be changed due to a violated specification, the outcome of this new design is uncertain. The uncertainty typically increases with the number and magnitude of changes to the known created design.

This chapter presents a method to incorporate the uncertainty of the design change into the change strategy, resolving some shortcomings from Chapter 4. The following sections improve the design change analysis described in 4.3, and modify the determination of the expected cost as described in 4.4. The analysis of the expected outcomes in 4.2 remains unchanged.

5.2 Likelihood of Actual Design Responses Occurring

In order to determine the overall cost of a design change, the likelihood of a certain set of actual design responses Y^* occurring has to be determined. The conditional uncertainty distribution $pdf^U(y_{k,l,j})$ represents the probability distribution of the actual design responses Y^* based on the predicted design responses Y. Figure 28 shows a conditional uncertainty distribution of the unchanged defective design as described in Chapter 4. Two possible design changes, design change A and design change B are shown with respect to the possible prediction uncertainties which could be resolved by means of these design changes.

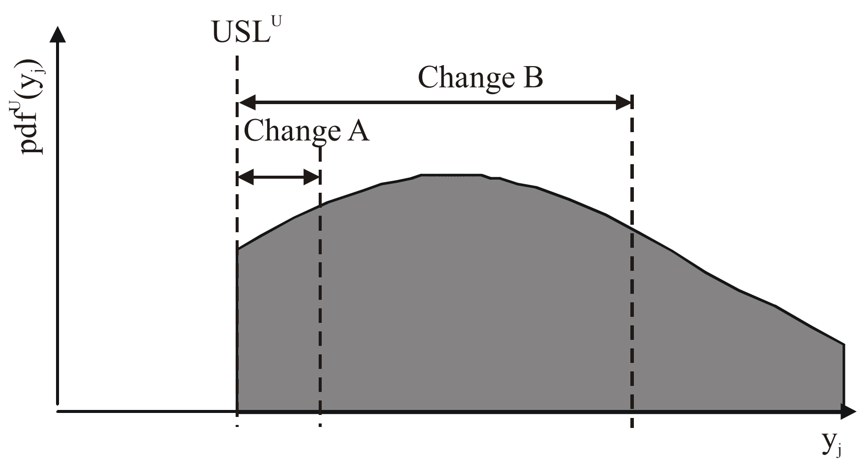


Figure 28: Conditional Uncertainty Distribution

5.3 Likelihood of Design Change Success

In order to accurately determine the likelihood of success for a given design change, the probability distribution of this design change being satisfactory would have to be known. Unfortunately, this distribution is very difficult to obtain. Due to lack of better methods, human estimation may be used to determine these uncertainty distributions. Within this chapter, the uncertainty distribution of the success of a design change is nominated as $pdf^{S}(y^{*}_{j})$, where y^{*}_{j} represents the actual mean design response measured in the design

Figure 29 plots the probability distribution $pdf^{S}(y^{*}_{k,l,j})$, which describes the likelihood of a design change resolving the defect. Please note that these distributions are truncated at the specification limit under uncertainty, as a non-violated specification is not defective.

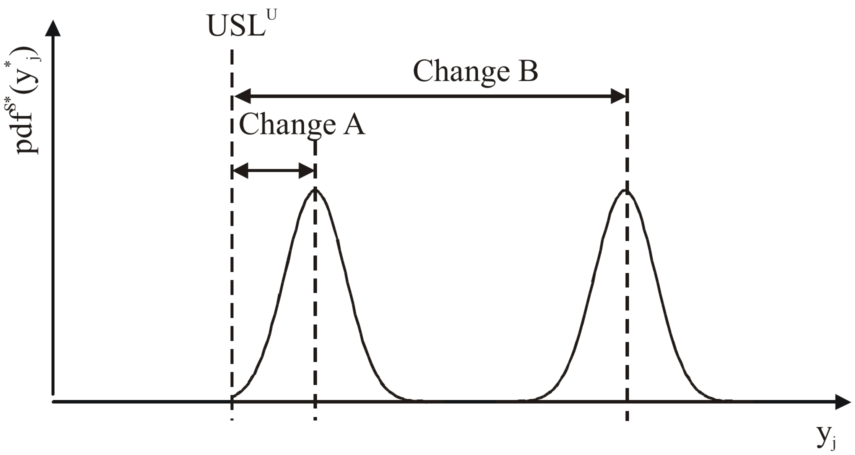


Figure 29: Probability Density Function of Successful Design Change

Since resolving a given defect would also resolve all smaller defects, the likelihood of resolving a given defect by means of a certain design change is represented by the cumulative distribution function as shown in Figure 30. The cumulative distribution function $cdf^{S}(y^{*}_{k,t,j})$ can be calculated as shown in Equation 71, determining the probability of resolving a defect with prediction uncertainty.

$$cdf^S\left(y^*_{k,t_l,j}\right) = 1 - \int_{USL^U}^{y_j} pdf^S\left(y^*_{k,t_l,j}\right) dy^*_{k,t_l,j} \quad if\ M_{k,t_l} = -1$$

$$cdf^S\left(y^*_{k,t_l,j}\right) = \int_{y_j}^{LSL^U} pdf^S\left(y^*_{k,t_l,j}\right) dy^*_{k,t_l,j} \quad if\ M_{k,t_l} = 1$$

Equation 71

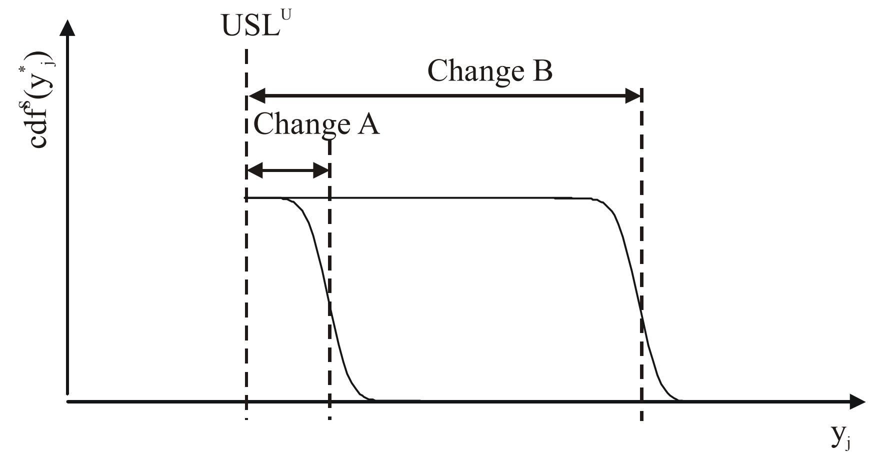


Figure 30: Cumulative Density Function of Successful Design Change

If more than one design response is evaluated, the joint probability of resolving the defect P^R_{k,t_l} for a given design change and a given expected outcome can be evaluated as shown in Equation 72. If a design variable violates a specification limit, the probability of resolving the defect is determined. If a design variable does not violate the specification limits under uncertainty, no resolving of the defect is necessary. Please note that the likelihood of a design change resolving a defect depends on the value of actual design responses.

$$P^R_{k,t_l} = cov + \prod_{j=1}^{n} \begin{cases} 1 & LSL^U_j \le y^*_{k,t_l,j} \le USL^U_j \\ cdf^S\left(y^*_{k,t_l,j}\right) & else \end{cases}$$

Equation 72

5.4 Design Change Strategy and Cost

Due to the uncertainty of a design change, it cannot be guaranteed that a design change will resolve a given defect. Rather, a likelihood of resolving a given defect can be determined based on the design change and the occurring uncertainty. If a design change is attempted, the change may be successful. The cost of a successful design change C^S_{k,t_l} consists of the cost of the changed part C^M_{k,t_l} and the cost of the change C^C_{k,t_l}, as described in Chapter 4 and shown in Equation 73.

$$C^S_{k,t_l} = C^M_{k,t_l} + C^C_{k,t_l}$$

Equation 73

However, if a design change is unsuccessful, a strategy is needed to determine the subsequent action. As previously described in the dissertation, the possible design changes for a given expected outcome are sorted according to the cost and likelihood of resolving the defect, with dominated designs reduced. If a design change is unsuccessful, the design change with the least total cost $C^{T}_{k,t_{l+1}}$ from the possible design changes with a larger likelihood of resolving the defect is selected. Therefore, the cost of an unsuccessful design change C^{U}_{k,t_l} consists of the design change cost for the attempted design change C^{C}_{k,t_l} and the total cost of the least expensive subsequent design change $C^{T}_{k,t_{l+1}}$ as shown in Equation 74.

$$C^{U}_{k,t_l} = C^{C}_{k,t_l} + Min\left[C^{T}_{k,t_{l+1}}, C^{T}_{k,t_{l+2}}, \ldots C^{T}_{k,t_{2^m}}\right]$$

Equation 74

Therefore, the total cost of a design change C^{T}_{k,t_l} consists of the cost of a successful design change C^{S}_{k,t_l} including the likelihood of resolving the defect P^{R}_{k,t_l} and the cost of an unsuccessful design change C^{S}_{k,t_l} including the likelihood of an unsuccessful design change 1- P^{R}_{k,t_l} for any given uncertainty as shown in Equation 75. Again, please note that the total cost of a design change depends on the value of actual design responses and the magnitude of the required change to move the design responses within the specification limits.

$$C^{T}_{k,t_l} = C^{S}_{k,t_l} \cdot P^{R}_{k,t_l} + C^{U}_{k,t_{l+1}} \cdot \left[1 - P^{R}_{k,t_l}\right]$$

Equation 75

The design change with the largest likelihood of resolving the defect needs special consideration. If this design change would fail to resolve the defect, then the cost of an unsuccessful design change would be the cost of the change and the cost of design failure as shown in Equation 76.

$$C^S_{k,t_{2^n}} = C^F + C^C_{k,t_{2^n}}$$

Equation 76

As the total cost of a design change C^T_{k,t_l} depends on the total cost of all subsequent design changes $C^T_{k,t_{l+1}}$, the evaluation of the total cost has to be done starting from the design change with the largest likelihood of success $P^R_{k,t_{2^m}}$, followed by all design changes with a smaller likelihood.

5.5 Expected Cost of an Expected Outcome

The total cost C^T_{k,t_l} for each design change as calculated above represents the expected cost of a certain design change, given a certain set of actual design responses. For any given set of actual design responses, always the design change with the least total cost is C^T_{k,t_l} selected. The total cost C^T_k of an expected outcome M_k for a given set of design responses is shown in Equation 77.

$$C^T_k = Min\left\lfloor C^T_{k,t_1}, C^T_{k,t_2}, \ldots C^T_{k,t_{2^n}} \right\rfloor$$

Equation 77

Therefore, the expected cost of the design change is the integral of the least total cost ${C^T}_k$ over the whole design space including the likelihood of specification violation for a given expected outcome as shown in Equation 78. The expected cost can then be used to improve design as described in Chapter 4.

$$C_k^E = \int_{LL}^{UL} C_k^T \cdot \left[\prod_{j=1}^{n} \begin{cases} 1 & \text{if } M_k = 0 \\ pdf^U(y_{k,l,j}) & else \end{cases} \right] dY$$

$$\text{where} \quad UL_j = \begin{cases} \infty & \text{if } M_k = -1 \\ LSL^U & f\ M_k = 1 \end{cases}$$

$$LL_j = \begin{cases} USL^U & \text{if } M_k = -1 \\ -\infty & f\ M_k = 1 \end{cases}$$

Equation 78

5.6 I-Beam Example

The flexible design methodology will be demonstrated using the robust design derived in Chapter 2 as the initial design shown in Table 23. The flexible design evaluation for this initial design using the deterministic design change assumption has been evaluated in Chapter 4.

Table 23: Initial Design Variables for Robust Design

Design Variable	Nom.	Value
Beam Height	x_1	34.8mm
Modulus	x_2	182,200 N/mm^2

5.6.1 Likelihood of Actual Design Responses Occurring

The probability density function of the actual design change occurring is shown below in Figure 31 based on the conditional uncertainty distribution determined in Chapter 4. Note that the distribution is truncated at the upper specification limit and scaled to ensure a total area of one.

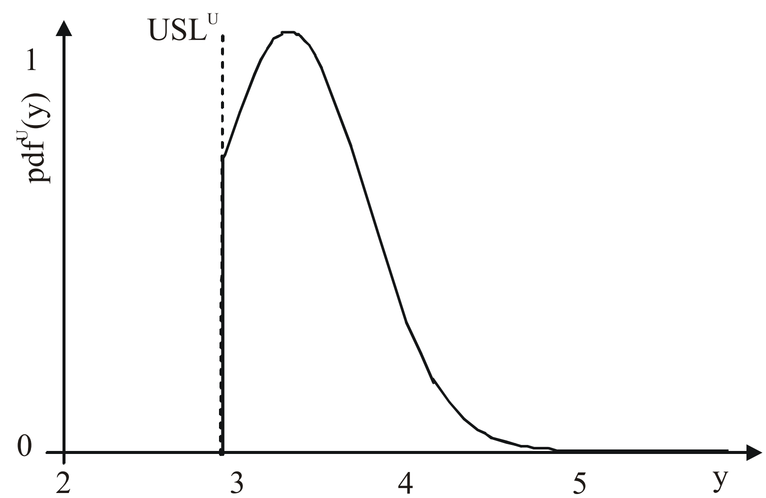


Figure 31: I-Beam Conditional Uncertainty Distribution of Deflection for Robust Design

5.6.2 Change Uncertainty Distribution

The uncertainty in the design change is assumed to have a standard deviation of 0.1 mm. The mean of the response distribution is the largest actual design response under uncertainty, which can be resolved using a discrete design change assumption. Figure 32 shows the probability density functions of the different design changes. Please note, that the function for changing the height and the function for changing the height and the modulus overlap.

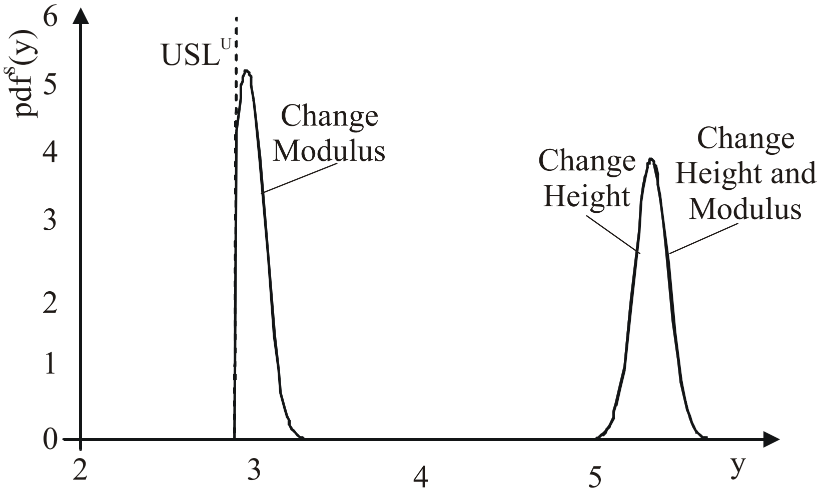


Figure 32: I-Beam Design Change Probability Density Function for Robust Design

Table 24 represents the deterministic ability to resolve a defect using the different design canges. In addition, the possible design changes are already sorted according to the ability to compensate for prediction uncertainties. The upper specification limit under uncertainty is at a value of 2.899 mm. A design change in modulus would be able to adjust for actual deflections up to 2.969 mm. A design change in height would be able to adjust for actual deflections up to 5.338 mm, i.e. if the actual measured deflection of the design would be 5.3mm, it would be possible to adjust for the uncertainty by changing the height in order to satisfy the quality requirement. The option not to change the design but to accept a design failure is possible for any design response. Figure 33 shows the cumulative density function representing the likelihood of resolving a defect for any given actual response using different design changes.

Table 24: I-Beam Design Changes for Robust Design

Change	Index	Deterministic Change (mm)
Modulus	S_3	2.969
Height	S_2	5.338
Height and Modulus	S_4	5.342
No Change	S_1	∞

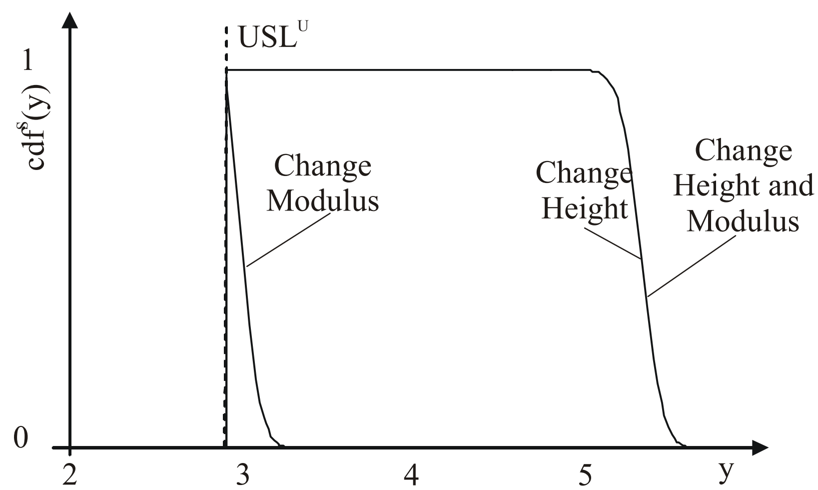


Figure 33: I-Beam Design Change Cumulative Density Function for Robust Design

5.6.3 Cost of Design Change for a given Defect

To develop a strategy regarding the selected design change, the expected cost of a design change has to be determined in order to allow a comparison with other design changes. The uncertainty distributions determine the likelihood of success for a given design change and a given magnitude of the defect. The cost of a successful design change is the cost of the part and the cost of the change. If the design change fails to resolve the defect, the strategy is to select a design change with a higher probability of success.

Therefore, the evaluation of the cost of a design change has to start with the most reliable design change. The most reliable design change for this example is the option not to change the design but to accept a design failure. This approach will always be the last option from the possible design changes at a cost of $5.00. The second most reliable design change is the option to change the height and the modulus. If the change is successful, the created cost consists of the part cost of $2.033 and the change cost of $0.301, giving a total cost of success of $2.334. However, if the change is unsuccessful, the only remaining option is a design failure. Therefore, the cost of an unsuccessful design change consists of the change cost of $0.301 of the attempt to resolve the defect and the failure cost of $5.00 of failing the design change, giving a total cost of an unsuccessful design change of $5.301. The expected cost of a change in the height and the modulus is shown in Figure 34, including the failure cost. It can be seen, that a design change using the height and the modulus is only justified economically for actual deflections of less than 5.5 mm. If the measured deflection exceeds 5.5 mm, then it is advisable not to change the design due to the small likelihood of success.

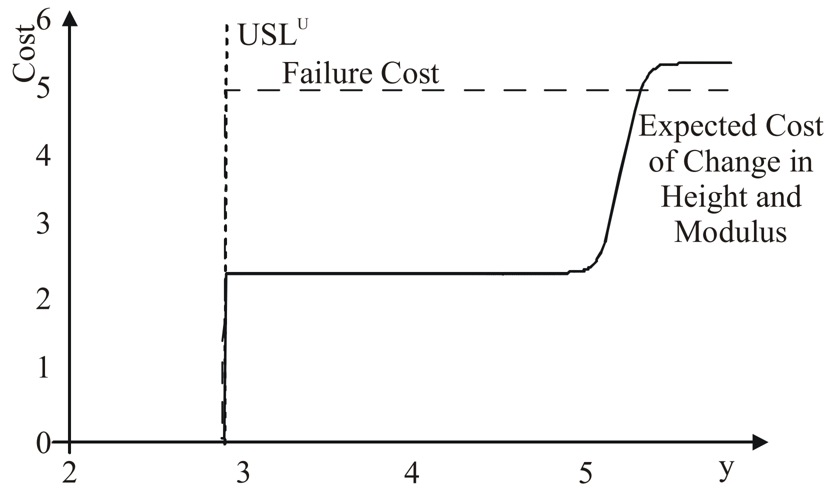


Figure 34: I-Beam Cost of Change in Height and Modulus for Robust Design

The next less reliable design change is a change in the height. The cost of success is the cost of the part of $2.029 and the cost of the change of $0.300, giving a total cost of success of $2.329. The cost of an unsuccessful design change is the cost of the change of $0.300 and the least expensive alternative option, i.e. either a change in height and modulus or a design failure. Depending on the actual deflection, this alternative and the cost of the alternative may change. Figure 35 shows the expected cost of a change in the beam height, as well as the expected cost of a change in the height and the modulus.

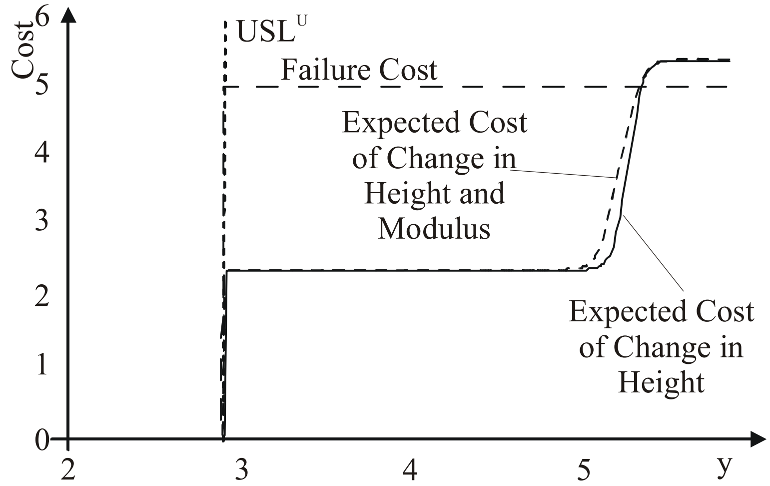


Figure 35: I-Beam Cost of Change in Height for Robust Design

The least reliable design change option is a change in the modulus. The cost of a successful change consists of the part cost of $1.649 and the change cost of $0.001, giving a total cost of $1.650. If the design change fails to resolve the defect, the alternative will be the least expensive other design options, a change in the height, a change in the height and the modulus, and a design failure. Figure 36 shows the expected cost of a change in the modulus, as well as the expected cost of other design changes.

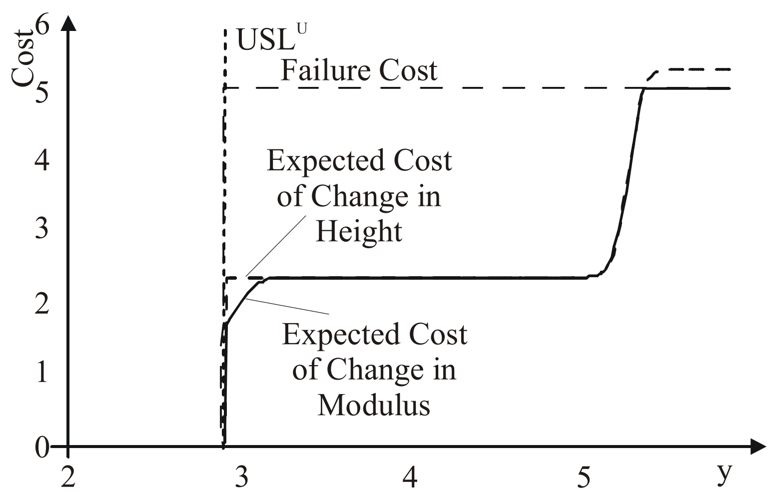


Figure 36: I-Beam Cost of Change in Modulus for Robust Design

5.6.4 Expected Cost of Design Change

The strategy for selecting the design change is to always select the change with the least expected cost for a given design response. Figure 37 shows the least expected cost for all design responses and the recommended strategy of design change. Please note, that it should be attempted to resolve a defect by means of a change in the modulus for deflections up to 3.27mm. At this point, there is a very small change of resolving the defect as shown in Figure 33. However, because the design change is so inexpensive, it is worth the attempt. If the change fails, the added cost is minimal, and a subsequent change of the height is attempted.

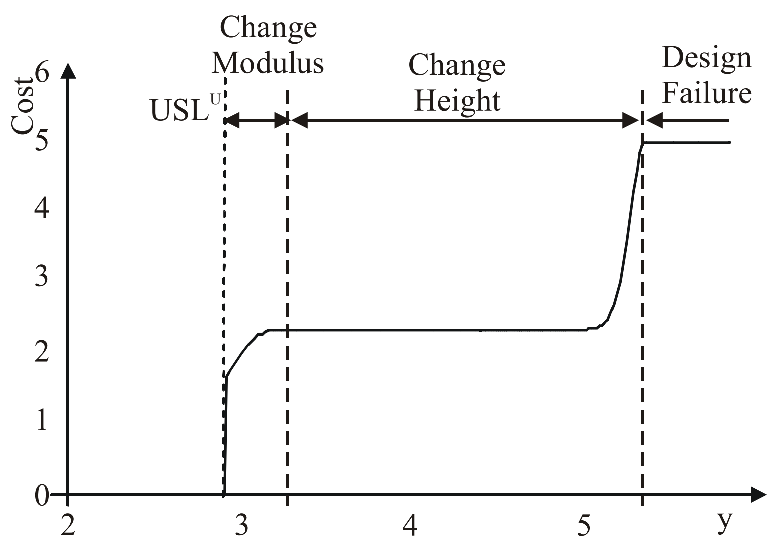


Figure 37: I-Beam Cost of Design Change for Robust Design

The least cost can now be combined with the likelihood of the defect occurring and integrated over the range of possible defects to determine the expected cost of resolving the expected outcome. In this example, the expected cost of resolving the defect was determined to be $2.26. This can now be combined with the cost of a non-defective design and the likelihood of a defect occurring into the expected cost of the design as shown in Table 25. The expected cost of the design was determined to be $2.145.

Table 25: Design Cost of Robust Design

Design Variable	Expected Cost of Expected Outcome	Probability of Expected Outcome	Expected Cost (Fraction)
No Defect	$1.645	19.2%	$0.315
Excessive Deflection	$2.26	80.8%	$1.826
		Expected Cost	$2.145

5.6.5 Flexible Design

The expected cost for the I-beam using the improved change methodology was minimized to determine the flexible design. The design variable values of this flexible design are shown in Table 26. The least cost for any defect is shown in Figure 38. It can be seen that a change in the modulus is attempted for a larger range of possible defects as compared to Figure 37. A change in modulus within the given range has a much larger likelihood to resolve the defect. In addition, for a certain range of defects it is most beneficial to utilize a change in height and the modulus as the first attempt to resolve the defect.

Table 26: Flexible Design Initial Design Variables

Design Variable	Nom.	Value
Beam Height	x_1	39.2mm
Modulus	x_2	162,000 N/mm^2

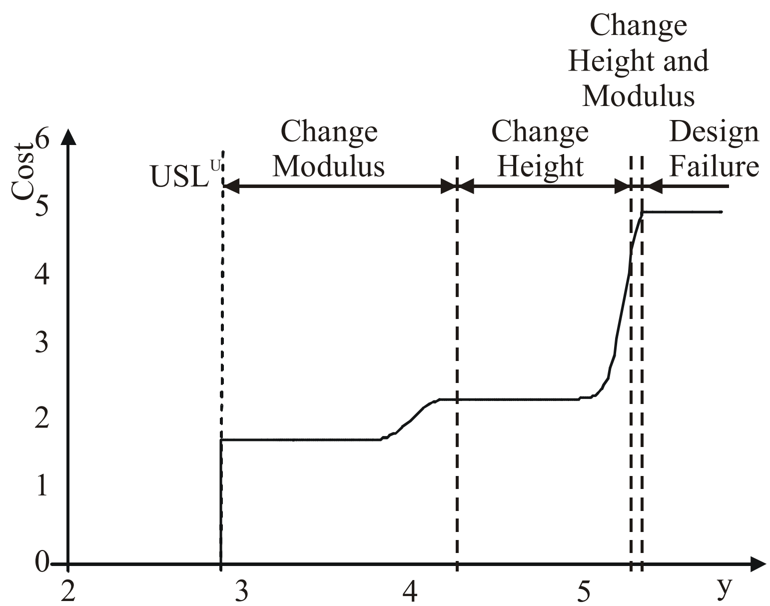


Figure 38: I-Beam Flexible Design Cost of Design Change

Table 27 shows the overall expected cost of the design. Although the initial part cost is increased compared to the robust design, the expected cost is significantly reduced by $0.43 from $2.15 to $1.72, representing a savings of over $215,000 for 500,000 parts. This is mainly due to the reduced likelihood and cost of a design change, where the flexibility of a change in the modulus was utilized and an expensive change in the beam height is frequently avoided.

Table 27: Design Cost of Flexible Design

Design Variable	Expected Cost of Expected Outcome	Probability of Expected Outcome	Expected Cost (Fraction)
No Defect	$1.716	67.0 %	$1.316
Excessive Deflection	$1.750	33.0%	$0.408
		Expected Cost	$1.724

5.7 Summary

The handling of uncertainty offers an improved method to utilize the flexible design methodology, where the likelihood of a successful design change and the cost of a design change is included in the strategy of selecting a given design change. An inexpensive design change, e.g. the modulus, is attempted even if there is a low possibility of the design change resolving the defect. If the change would be successful, it would have been possible to avoid an expensive design change. If the change is unsuccessful and fails to resolve the defect, no significant cost is added. This method of handling uncertainty in a design change is improved compared to the deterministic handling of design changes as described in Chapter 4. However, the problem in this approach lies in obtaining the necessary estimates regarding the uncertainty of the design change. As this information is frequently unavailable or can be obtained only under great efforts, a deterministic design change evaluation may be preferable. Therefore, Chapters 6 and 7 containing the value of information and a complex industry example will utilize the deterministic design change analysis.

CHAPTER 6

VALUE OF INFORMATION

6.1 Introduction

Information about the relation between the design variables and the design responses in engineering design is a valuable asset in the design process. Within this dissertation, design information is seen as the ability to predict the behavior of an engineering design. This ability is necessary in order to select a feasible design.

If the functional relations between the design variables and the design responses would be known without any uncertainty, it would be possible to create a design having exactly the desired responses. Unfortunately, this information is usually not free but has to be discovered and validated. This process of investigating the design requires human, financial, and material resources, and therefore creates cost.

Statement 1: Acquiring information requires resources.

In addition, information regarding an engineering design is not always perfect. While not necessarily wrong, the information might be inaccurate and uncertain, causing the behavior of the physical embodiment of a design to differ from the predicted design behavior. This difference can cause additional defects and reduce the value of the design. The sources of this incorrect and therefore uncertain information are described in Chapter 3.

Statement 2: Information is uncertain

A schematic relation between the cost of the information and the defect change cost is shown as a function of prediction error in Figure 39. A reduced prediction error reduces the cost due to design changes and overly conservative safety factors. However, in order to reduce the prediction error, effort is required to gain information in order to improve the accuracy of the prediction. This cost of information generally increases as the prediction error decreases.

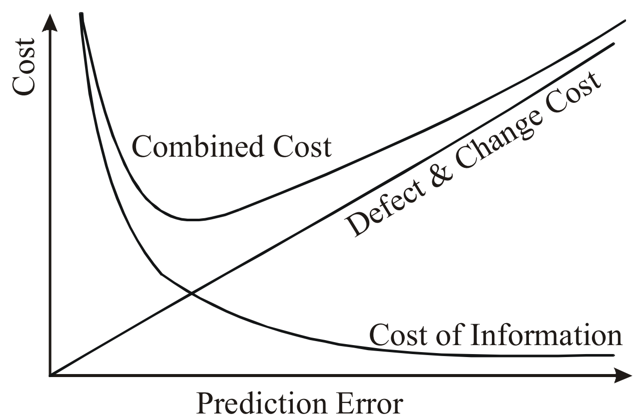


Figure 39: Schematic Cost as a Function of the Prediction Error

The method described in this chapter aims to determine a trade-off between the cost of information and the value of information. An outline is shown in Figure 40. The considered prediction models are determined and the cost of information is estimated. An economic prediction model is created to simulate the more accurate prediction models using the flexible design methodology. The benefit of the prediction models is compared to the cost of the prediction models, and the model with the best trade-off is created. Using this model, the design with the least expected cost is determined using the flexible design methodology, and this design is then applied to production.

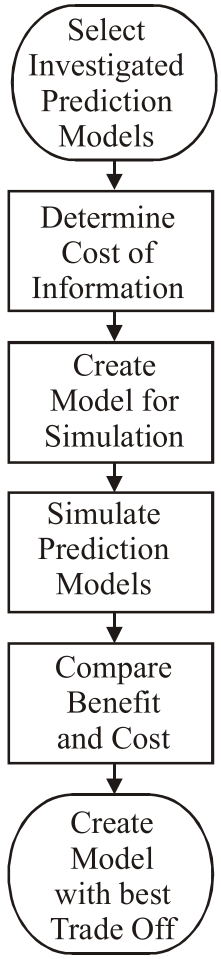


Figure 40: Value of Information Method Outline

The value of information has been analyzed for human experience, where a more experienced engineer will usually perform better than a less experienced engineer. (Ahmed et al. 1999) discusses the relation between data and information by studying the different tools and methods used by designers of different levels of experience. (Lowe et al. 1999) also investigates the organization and use of information by design teams with different levels of experience. (Wood et al. 1998) describes an approach to formalize human experience of previous design projects in order to use this experience for the improvement of current design projects. (Yoshimura and Kondo 1997) emphasizes the importance of groups to pool human experience in engineering design. (Jahangir and Frey 1999) describes methods to measure information content in order to provide a basis to determine the independence of information for engineering design. (Kelly 1956) describes information rates on a very theoretical basis.

6.2 Cost of Information

In order to gain information about the design, resources have to be invested. Human estimation can be used to predict the behavior of a design. However, the prediction accuracy varies wildly depending on the experience of the design team. Therefore, this method is rarely used by itself but rather to support and simplify other more scientific methods. The cost of this method depends on the cost of the human resources and the time required to estimate the design performance.

Another frequently used method to estimate the behavior of the design is the creation of analytical models. These models are usually very accurate if the analytical model describes the design completely. Prediction accuracy is lost as the number and effect of bad assumptions increases. The cost of this information generating method also depends largely on the cost of the human resources and the time required to create the analytical model.

Related to the analytical model is the finite element method. The accuracy of this method also depends on the number and effect of assumptions. Although it is possible to use the finite element method to estimate the functional relation across the whole design space, this would require the full analysis of every design of interest during the design development process. To reduce the computational expense, the performance data can be interpolated or extrapolated from a response surface model. The cost of this method depends not only on the cost of human resources, but also on the necessary computational equipment and software.

Empirical data is also commonly used in engineering design. The problem of empirical data is the estimation of the design behavior between and outside of the conducted designs. Interpolations and extrapolations are used to estimate the design behavior for the complete design space. This method is also used to measure the actual design responses by testing actual instances of the design. The cost of this method is rather large, depending not only on the human resources but also on the effort in material and processing to create the empirical data. This also increases with the number of measured sample points, which frequently reduces the error of the prediction model.

The cost of generating the information is nominated as C^I. Note that 1) the available methods to acquire information depend on the design system and the available resources, and 2) the quality may vary depending on numerous influences like the experience of the design team or the quality of the finite element software. Therefore, no general evaluation of the cost of the information C^I is presented in this dissertation. Cost accounting methods have to be used with respect to the different information generating methods. For an overview about cost accounting, please refer to (Wöhe 1993). The following section describes the required information and assumptions to evaluate the value of information.

6.3 Requirements

In order to compare the expected cost for different model accuracies, it is possible to use multiple prediction models with different prediction accuracies. However, to do this, both models have to be created in order to compare them with each other. Yet, if there exist different models with different degrees of accuracy, it is common sense to use the most accurate model. A comparison would merely prove that the less accurate model has a lower value.

However, it is desirable to know the value of a prediction model before investing the time and resources of creating the prediction model. A prediction of the effect of increased model accuracy on the expected cost has to be made. To perform this prediction, an initial design model has to exist. This initial design model has to represent the basic relations between the design variables and the design responses, and to include all design variables and design responses utilized in the more complex models. This model is used to estimate the predictions of more accurate models to evaluate the value of information.

Theoretically, this model can be used to compare prediction models with less and more accuracy than the initial design model. However, if the initial model is created, it will be of little use to simulate a less accurate model. Therefore, the simulated models should have higher prediction accuracy than the initial model used. In addition, once the effort for creating the initial model is spent, it is impossible to reallocate the consumed resources. Therefore, this initial model should be inexpensive to keep the initial costs low. In summary, a low cost model should be used to simulate other more expensive and more accurate models.

The initial model is used to compare the value of different more accurate models. The more accurate models do not have to be created. However, the prediction error distribution for the different responses would have to be estimated to evaluate the value of information. Unfortunately, as these models are not yet created, the prediction error distribution cannot be measured. Previous experience with similar models can be used to estimate the uncertainty of a prediction model. Human experience may also be used to estimate the prediction accuracy based on prior experience with similar models. (Hunkar 1982; Hunkar 1987; Hunkar 1991; Hunkar 1998a; Hunkar 1998b) uses class factors to group injection molding machines according to their process repeatability, i.e. their production accuracy. (Glaeser et al. 1995) analyzes mathematical techniques for uncertainty and sensitivity analysis, concluding that if an actual model is available it should be used and not simplified by fitting a prediction equation on the model due to the increased uncertainty.

The simulation of the effects of uncertainty of a more accurate model using a less accurate model predicts the different expected outcomes and design changes due to uncertainty. Note that this is only used to compare different levels of uncertainty. It does not provide more accurate design predictions! The resulting optimal designs for a simulated uncertainty can only be used to evaluate the effect of uncertainty towards the design. It must not be implemented as a "better design". In order to receive the benefits of the reduced uncertainty, the improved model has to be created and the flexible design method has to be performed using the improved model.

6.4 Value of Information

To determine the value of information of different design predictions, the design is optimized for the expected cost using different prediction uncertainties. First, the cost of the information C^I_u for different uncertainty assumptions has to be determined and the resulting uncertainty distributions $pdf^U_u(Y)$ estimated. Table 28 shows an example of different prediction models, including cost C^I_u and uncertainty distributions $pdf^U_u Y)$.

Table 28: Prediction Models

r	Cost	Uncertainty	Comments
1	C^I_1=0	$pdf^U_1(Y)$	e.g. Analytical Model
2	C^I_2	$pdf^U_2(Y)$	e.g. Empirical Data
3	C^I_3	$pdf^U_3(Y)$	e.g. FEM Analysis

As mentioned above, an initial model is required to simulate the behavior of the design under uncertainty. Therefore, the effort for creating this model has to be spent before determining the value of information. Subsequently, no new cost is added for a later creation of the model, and the cost of information C^I_1 is set to zero for the purpose of the uncertainty evaluation.

6.4.1 Determine Ideal Designs

The expected cost C^E_u is minimized using the uncertainty $pdf^U_u Y)$ of the design model u for the initial prediction model. Subsequently, designs are determined by minimizing the expected cost using the different uncertainty assumptions. For each uncertainty assumption j one design X_u is created, representing a desired expected cost within the design space with respect to the described prediction uncertainty.

$$\begin{aligned} &Min\left(C_u^E\right) \qquad \forall u \\ &using\ pdf_u^U(Y) \end{aligned}$$

Equation 79

6.4.2 Compare Designs

It is now possible to compare one or more different prediction models using the flexible design methodology to evaluate the expected cost as shown in Table 29. Again, please note that only future cost is considered. As the prediction model used to simulate the prediction accuracy is already created, no further cost is added, and the cost of information for this model is zero.

Table 29: Uncertainty Comparison

Uncertainty	Expected Cost	Cost of Information
$pdf^U_1(Y)$	C^E_1	$C^I_1=0$
$pdf^U_2(Y)$	C^E_2	C^I_2
$pdf^U_3(Y)$	C^E_3	C^I_3

The amortized cost of retrieving the information C^I can then simply be added to the expected cost C^E in order to represent the expenses of the design C^{EI} including possible design changes and the cost of retrieving the information as shown in Equation 80. Note that V represents the production volume, as the cost is estimated per part. This generates multiple designs with different costs including the expected cost and the cost of information.

$$C_u^{EI} = C_u^E + \frac{C_u^I}{V}$$

Equation 80

This information can now assist in selecting and developing a desired prediction model, giving the best trade-off between the benefits of the improved prediction accuracy and the cost of retrieving the information. The prediction model with the least cost C^{EI} represents the best trade-off assuming a risk indifferent approach. However, if the design team is willing to take the risks of using a more economic but less accurate model, it might choose a model where the cost of information is less than the optimal trade-off, with an subsequently larger expected cost. If a risk adverse design approach is utilized, a more accurate design model might be selected even if the benefits of the accuracy do not justify the added cost of the information. In this case, there might be other benefits not considered in the flexible design methodology, as for example a reduction in the time needed for possible design changes.

6.5 I-Beam Example

The method to determine the value of information will be demonstrated using the previous I-beam example.

6.5.1 Prediction Models

The value of information will be compared for the I-beam using two different prediction models, both with a standard normal distributed prediction uncertainty. The first prediction model is the set of response surface equations developed in Chapter 2. The deflection prediction using the response surface model has a mean prediction error of 0.4mm and a standard deviation of the prediction error of 0.452mm under uncertainty, mainly due to the errors in fitting a quadratic equation to the sampled data points. The second prediction model will be the analytical model, also described in Chapter 2. The error of the deflection for the analytical model due to uncertainty has a mean error of zero and a standard deviation of 0.05mm stemming from parametric variation. An overview of the uncertainty distributions is shown in Table 30.

Table 30: I-Beam Prediction Error Distributions for the Deflection

Prediction Model	Mean (mm)	Standard Deviation (mm)
Analytical Model	0.00	0.05
Response Surface Model	0.40	0.45

6.5.2 Cost of Information

To generate either of these two prediction models, effort has to be invested in researching and organizing the information. This effort will differ for the different prediction models, causing the cost of the information to differ. For the example, of the I beam, the generation of the analytical model and the prediction model can both be done quickly with a low cost and resource requirement.

However, to demonstrate the methodology, the two prediction models will possess different costs, where the more accurate model is more expensive than the less accurate model. Although this is not necessarily true for the I-beam example, it is valid for numerous industry examples, where for example the number of experimental sample points in a design of experiments increases accuracy and cost, or where the cost of human resources is increasing with the number and experience of the project engineers.

Therefore, for this example, the cost of creating the prediction model is assumed to be $1,000 for the response surface model and $4,000 for the analytical model. An overview of the cost of information is shown in Table 31, where the overall cost of information and the cost of information per part for 50,000 parts are given.

Table 31: I-Beam Cost of Information

Prediction Model	Cost of Information	Cost of Information per Part C^I
Analytical Model	$4,000	$0.08
Response Surface Model	$1,000	$0.02

In order to compare different prediction models, the less expensive response surface model is used to provide a basis to evaluate the value of information. Therefore, the efforts of $1,000 for creating the response surface model have to be invested, and subsequently there will be no additional cost for the creation of the model. The objective is to determine if it is more economic to utilize the existing response surface model, or if it is more economic to create the more accurate analytical model at the additional cost of $4,000.

6.5.3 Flexible Design Evaluation

To compare the two prediction models, a flexible design evaluation has to be performed, creating an expected cost for each prediction uncertainty. The response surface model will be used to simulate the prediction accuracy of the analytical model. The following sections describe the designs with the least expected cost for the different prediction uncertainty distributions.

6.5.3.1 Response Surface Model

The design with the least expected cost for the response surface model was evaluated in Chapter 4. The resulting design from the flexible design methodology is shown in Table 32.

Table 32: Initial Design with Least Expected Cost for Response Surface Model

Expected Cost	$1.75
Beam Height	40.9mm
Modulus	149,000 N/mm^2
Probability of No Change	70.2%
Probability of Any Change	29.8%
Probability of Failure	0.00%
Probability of Changing Height	2.1%
Probability of Changing Modulus	27.7%

6.5.3.2 Analytical Model

The effects of the prediction accuracy of the analytical model were simulated using the response surface model, but with lower mean and standard deviation of the prediction uncertainty of the analytical model. An overview of the results is shown below in Table 33. Using the reduced uncertainty distribution of the analytical model, it was possible to reduce the expected cost from $1.75 to $1.65. This reduction in cost comes primarily from a reduction in the beam height. This reduction was possible due to a reduced change likelihood stemming from the smaller prediction error. Therefore, the design requires less robustness against uncertainty, and the beam height can be reduced without a significant increase in the probability of a design changes.

Table 33: Initial Design with Least Expected Cost for Analytical Model Uncertainty

Expected Cost	$1.65
Beam Height	35.1mm
Modulus	181,000N/mm2
Probability of No Change	86.1%
Probability of Any Change	13.9%
Probability of Failure	0.00%
Probability of Changing Height	0.21%
Probability of Changing Modulus	13.9%

It is important to point out that this analysis is only used to compare the effect of model accuracy, and it is not recommended to build this design. If this analytical prediction model is desired to choose a design, the analytical model has to be developed and the flexible design methodology has to be performed using the actual analytical model.

6.5.4 Value of Information

Using the flexible design methodology to simulate the benefits of the improved prediction accuracy, it was determined that the analytical model would reduce the expected cost by $0.10 per part. Therefore, it would be beneficial to invest up to $0.10 per part into the development of the analytical model to harvest the benefits of the improved prediction accuracy. If the creation of the analytical model would cost more than $0.10 per part, an investment would be not advisable for a risk indifferent design approach. However, if the design team is willing to have additional expenses to reduce the risk of a design change, a cost of the model in excess of $0.10 may still be desirable. The cost of failure did not affect the design decision, as this design has a very low probability of failure. The probability of failure, i.e. the probability of a major redesign may also affect the decision regarding the use of a prediction model.

However, in this case, the analytical model was determined to cost $0.08 per part, and therefore it is recommended to invest the resources into the development of the analytical model, assuming a risk indifferent approach.

6.5.5 Flexible Design Evaluation for Analytical Model

The evaluation of the value of information justified the development of the analytical model on an economic basis. Therefore, the flexible design methodology uses the analytical model in order to determine the least expected cost is shown below in Table 34. It can be seen that the usage of the analytical model would reduce the expected cost even further, not only from $1.74 to $1.65 as predicted but to $1.61, justifying the expense of developing of the analytical model. There is a low probability of any design change, and if a defect occurs, it can be resolved by changing the modulus alone. It should be noted, moreover, that the flexible design method does not provide a guarantee of no change, but derives an optimal point at lower cost that may require a simple design change.

Table 34: Flexible Design using Analytical Prediction Model

Expected Cost	\$1.61
Beam Height	33.8mm
Modulus	172,000N/mm^2
Probability of No Change	90%
Probability of Any Change	10.0%
Probability of Failure	0.0%
Probability of Changing Height	0.0%
Probability of Changing Modulus	10.0%

6.6 Summary

The flexible design methodology can be utilized to determine the value of an accurate prediction model before creating the model by simulating the prediction accuracy using a less accurate model. The least expected cost can be determined for different model accuracies, and the benefit of improved accuracy can be measured. The comparison of the benefit with the cost of the information aids the design team in the selection of the utilized prediction model, and a strategy for the design development process can be chosen.

CHAPTER 7

APPLICATION EXAMPLE

7.1 Introduction: Thin Wall Monitor Housing

The flexible design methodology will now be demonstrated for an industry application. An injection-molding monitor housing was selected due to the complexity of the underlying relations and the significant effect of the processing variables towards the design responses. Since the information regarding the design change accuracy is not available, the deterministic design change analysis as described in Chapter 4 was utilized. For detailed information regarding the prediction models please refer to Appendix D.

7.1.1 Design Variables

The prediction models use a significant number of continuous and discrete design variables to predict the properties of the monitor housing. These variables are listed below in Table 35, including a brief description of the design variables.

Table 35: Design Variables

Design Variable	Type	Unit	Comment
Melt Temperature	Continuous	ºC	Temperature of plastic material
Mold Temperature	Continuous	ºC	Temperature of mold
Eject Temperature	Continuous	ºC	Required temperature for ejection
Injection Time	Continuous	s	Duration of injection cycle
Thickness	Continuous	mm	Average wall thickness of housing
Flow Length	Continuous	cm	Distance from gate to furthest corner
Material Type	Discrete	#	Resin grade
Number of Tools	Discrete	#	Number of production tools used
Availability	Continuous	hr/week	Time for production per week
Projected Area	Continuous	cm^2	Projected area of the housing
Production Volume	Discrete	#	Number of produced parts

Four different material types are commonly used for the monitor housing. Three materials consist of a polycarbonate (PC) and acrylonitrile-butadiene-styrene (ABS) resin blend. An additional material consists of a polycarbonate (PC) and polystyrene (PS) resin blend. Three of these materials are commercially available from two major resin suppliers and an additional fourth material is a non-commercial experimental grade from a major resin supplier.

If all available design variables were included in the flexible design analysis, a total of up to 331,776 design evaluations would be required. If each evaluation would take only one second, the complete flexible design analysis would require almost 4 days.

Therefore, only the most significant design variables are used within the flexible design analysis as determined by a main effects analysis. Other variables remain at their nominal value. Table 36 shows the investigated design variables including the constraint limits and the standard deviation of the noise. Table 37 shows the other design variables with a constant value, including the standard deviation of the noise.

Table 36: Investigated Design Variables

Design Variable	Type	Unit	Noise Deviation	LCL	UCL
Mold Temperature	Continuous	ºC	5	50	70
Thickness	Continuous	mm	0.1	1.5	3.5
Number of Tools	Discrete	#	n/a	3	4
Material Type	Discrete	#	n/a	1	4

Table 37: Constant Design Variables

Design Variable	Type	Unit	Noise Deviation	Value
Melt Temperature	Continuous	ºC	3	270
Eject Temperature	Continuous	ºC	3	90
Injection Time	Continuous	s	0.1	1
Flow Length	Continuous	cm	0	29
Availability	Continuous	hr/week	0	100
Projected Area	Continuous	cm^2	0	1,500
Production Volume	Discrete	#	0	500,000

The selection of the investigated design variables was based on the significance of these design variables toward the expected cost using the flexible design methodology. However, the impact on the design response is not the only criteria for selecting the design variables. The flexible design methodology aims to achieve a compromise between the cost of the part and the cost of the design changes. This compromise is necessary if the change of a design variable affects the cost of the part and the cost of the design changes inversely, i.e. changing the variable in one direction increases one cost and reduces the other. If a change in a design variable affects both costs in the same way, i.e. an increase of a design variable both reduces the cost of the part and the cost of changes, no trade-off is necessary and the variable is set to the nominal value giving the least part cost and the least change cost. Subsequently, a variable requiring no trade-off between the change cost and the part cost does not have to be analyzed by the flexible design methodology.

For example, the melt temperature has a significant effect on the performance and the cost of the product. However, there is little or no trade-off, since an increase in melt temperature reduces both the part cost due to a reduction in the required machine cost and the change cost due to a reduction of the melt pressure. Therefore, the melt temperature is set at the upper constraint limit and not investigated by the flexible design analysis.

7.1.2 Design Responses

Several design responses are evaluated to determine the feasibility of the design and to create the objective function for the design automation. Detailed information can be found in Appendix D. However, only four design responses are specified within the monitor-housing example presented within this chapter. These design responses are listed below in Table 38 including the related specification limits. The prediction error distribution of these design responses is assumed to be standard normal distributed with a mean of zero and a standard deviation as shown in Table 38. In addition to these specified design responses, the marginal part cost is also evaluated in order to provide an objective function. An error transformation was used to predict the response noise distribution based on the noise distribution of the design variables.

Table 38: Design Responses

Design Response	Unit	Uncertainty Deviation	LSL	USL
Melt Pressure	MPa	50	-	150
Shrinkage	%	0.03	0.01	0.3
Clamp Tonnage	t	100	-	700
Production Period	Weeks	2	-	18
Marginal Part Cost	$	n/a	-	-

7.2 <u>Transfer Functions</u>

The deterministic relation between the design variables and the design responses was established using a variety of different models and simulations. To reduce the computation time, some of the more computationally expensive simulations were approximated with a response surface prediction equation using a quadratic design of experiments. In order to estimate the response distribution based on the distribution of the design variables, a moment matching method has been applied as described in the appendix. Detailed information can be found in Appendix D.

7.3 Robust Design against Noise

The monitor housing design system has been optimized to reduce the marginal part cost with respect to the quality requirement as described in Chapter 2. Within this example, the mean response is required to be at least three standard deviations away from the specification limits. For this α value of three, the design with the least marginal part cost is shown in Table 39. The related design responses are shown in Table 40, including the standard deviation of the responses due to noise and the distance of the mean response to the specification limit (measured in number of standard deviations).

Table 39: Design Variables for Least Marginal Part Cost

Design Variable	Value
Mold Temperature	69.34
Thickness	1.948
Number of Tools	3
Material Type	4

Table 40: Design Responses for Least Marginal Part Cost

Design Response	Value	Noise Deviation	Distance to $\{LSL^U, USL^U\}$
Melt Pressure	49.9	13.94	{n/a, 7.17}
Shrinkage	0.140	0.002	{46.1, 56.4}
Clamp Tonnage	379.8	106.14	{n/a, 3.01}
Production Period	12.88	0.851	{n/a, 6.02}
Marginal Part Cost	5.949	n/a	n/a

7.4 Flexible Design Methodology

For the above design system, the flexible design methodology is applied. Two designs are compared with each other. The first design represents the robust design from section 7.3. This design is compared with an improved design, having the optimal expected cost.

For both cases, a design change cost matrix is used to evaluate the change cost of a design change. The change costs for the design variables was developed from interviews. As shown in Table 41, the design variables can be grouped into two categories. The number of tools and the thickness are very expensive changes. With respect to the overall cost, it is preferable to change inexpensive design variables if possible, e.g. the mold temperature and the material type require only a minimal change cost. Therefore, it is preferable to use these variables for design changes in order to improve the flexibility of the design. Within the following flexible design analysis it will be shown that a design with a small expected part cost is very likely to use these inexpensive design variables to adjust for possible prediction errors, and the change of expensive design variables is avoided whenever possible.

Table 41: Design Variables Change Cost

Design Variable	Change Cost	Change Cost per Part
Mold Temperature	$100	$0.00020
Thickness	$80,340	$0.16
Number of Tools	$200,220	$0.40
Material Type	$190	$0.000380

7.4.1 Optimal Robust Design

The design with the least marginal part cost may violate the quality requirements due to the prediction uncertainty. Table 42 shows the likelihood of the initial design violating the quality requirements for each design response, subsequently requiring a design change. It can be seen that the design is very likely to violate the clamp force requirements, causing an excessive number of defects. There is also a smaller probability of violating the production time and melt pressure requirements.

Table 42: Probability of Specification Violation under Uncertainty

Design Response	Probability
Melt Pressure	12.2%
Shrinkage	0.0002%
Clamp Force	49.3%
Production Time	9.9%

The expected cost of the design with the least marginal part cost is $7.34 as shown in Table 43. This is mainly due to the large number of design changes, where the unchanged design has only a probability of 40% of being feasible. Out of the 59% likelihood of design changes, the vast majority of defects require both a change of the wall thickness (42%) and the number of tools (48%). Unfortunately, the increase of the number of tools is the most expensive design change with a cost of $200,220, or $0.40 per part. The second most expensive design change is the change of the wall thickness, requiring costly retooling of $80,340, or $0.16 per part.

Moreover, there is only a 2% likelihood of resolving the defects with a fast and economic change of the material or the mold temperature. In summary, this design has a large likelihood of a design change, requiring the change of the two most expensive design variables investigated. In addition, there is a 0.8% chance of encountering a defect that cannot be adjusted for within the given analysis, creating an additional failure cost.

Table 43: Flexible Design Analysis Summary for Least Marginal Part Cost

Expected Cost	$7.34
Probability of No Change	40.1%
Probability of Any Change	59.1%
Probability of Failure	0.8%
Probability of Change in Mold Temperature	4.6%
Probability of Change in Thickness	41.7%
Probability of Change in Number of Tools	47.7%
Probability of Change in Material Type	11.4%

7.4.2 Optimal Flexible Design

By using the flexible design methodology, the behavior of the design with the least marginal part cost was improved to reduce the probability of a design change and to facilitate the flexibility of design change. This was achieved by increasing the wall thickness to improve the ease of flow and therefore reduce the required clamp tonnage. However, this also increased the material cost and the time required for cooling the part. To reduce the production time, the mold temperature has been decreased. The design variable values for the design with the least expected cost are shown in Table 44, which can be compared to Table 39.

Table 44: Design Variables for Least Expected Cost

RSM Flexible Design	Value
Mold Temperature	52
Thickness	2.12
Number of Tools	3
Material Type	4

This design reduces the probability of violating the clamp force requirement from 50% to 9% as shown in Table 45. The likelihood of violating the requirements on the melt pressure and the production time has also been reduced 7% and 1%, respectively. This design significantly increased the marginal part cost by $0.31 from $5.94 to $6.25. However, the expected cost including design changes has been reduced by $0.94 from $7.34 to $6.40 per part. This would reduce the overall cost of all parts by $470,000 from $3,670,000 to $3,200,000 due to the improved trade-off between the marginal cost of the part and the likelihood and cost of design changes.

Table 45: Improved Probability of Specification Violation under Uncertainty

Design Response	Probability
Melt Pressure	6.55%
Shrinkage	0.014%
Clamp Force	9.15%
Production Time	1.17%
Marginal Cost	$6.25

Table 46 shows a comparison between the design with the least marginal part cost in the previous section and the design with the least expected cost. The overall probability of the initial design requiring no design change has been more than doubled from 40% to over 83%. Subsequently, the probability of any change has been reduced to one fourth of the previous value. Out of the 15% likelihood of design changes, 8% can be resolved by changing only the melt temperature, a very inexpensive and fast design change. The likelihood of changing the two most expensive design variables, the number of tools and the wall thickness, has been extremely reduced. Overall, the probability of a design change has been greatly reduced, and out of these design changes, a large fraction can be easily resolved using the flexibility of the design.

Interestingly, the probability of failure has been slightly increased. This probability stems from an inability to simultaneously satisfy mutually exclusive specifications given the current statue of knowledge. There are different methods to reduce the probability of failure or to improve a design if a failure occurs. One option is to expand the design space, for example by considering a wall thickness in excess of 3.5mm or investigate additional high quality materials. Another option is to include design variables not previously considered in the flexible design methodology, as for example the flow length or the melt temperature. It is also possible to redesign the part, for example by adding flow channels to reduce the injection pressure or by breaking up the monitor housing into two smaller, less complex parts.

Table 46: Flexible Design Analysis Comparison

Design Element	Robust Design	Flexible Design	Change
Expected Cost	$7.344	$6.396	- $0.948
Probability of No Change	40.1%	83.8%	+ 43.7%
Probability of Any Change	59.1%	14.8%	- 44.3%
Probability of Failure	0.8%	1.3%	+ 0.5%
Probability of Change in Mold Temperature	4.6%	14.0%	+ 9.4%
Probability of Change in Thickness	41.7%	5.9%	- 35.8%
Probability of Change in Number of Tools	47.7%	1.8%	- 45.9%
Probability of Change in Material Type	11.4%	2.0%	- 9.4%

7.5 Value of Information

The value of information is determined by comparing two different prediction models with different prediction uncertainties. The first model is the response surface model as evaluated above. This model will be utilized to simulate the prediction accuracy of the second model. The second model utilizes simulations and finite element methods, having a significantly reduced uncertainty. The response surface model (RSM) and the finite element model (FEM) are compared in Table 47. The cost of creating the finite element model for the design space is assumed to be $100,000 or $0.20 per part. The cost of creating the response surface model is assumed to be $20,000 or $0.04 per part. However, since the response surface model is already created, there is no additional cost, therefore the cost of information for the response surface model is zero.

Table 47: Design Response Uncertainty Comparison

Design Response	Unit	RSM Deviation 1	FEM Deviation 2
Melt Pressure	MPa	50	40
Shrinkage	%	0.03	0.01
Clamp Tonnage	t	100	50
Production Period	Weeks	2	0.6
Cost of Information per Part	$	0.00	0.20

The design with the optimal expected cost for the response surface method has been evaluated in 7.4.2. The design with the least expected cost of the finite element model is simulated using the response surface model and the uncertainty distributions of the finite element model, resulting in the design summarized in Table 48.

Table 48: Design Variables for FEM Flexible Design

Design Variable	Value
Mold Temperature	53
Thickness	2.03
Number of Tools	3
Material Type	4

The two designs, referred to as RSM design and FEM design are compared in Table 49. The RSM design has an expected cost of $6.39, and the FEM design has an expected cost of $6.14. The difference in the expected cost represents the value of information of the FEM design of $0.25, exceeding the cost of information of $0.20. Combining the expected cost and the cost of the information, the FEM design with $6.34 is still $0.05 less expensive than the RSM design with $6.39. This would represent a savings of $25,000 for all parts due to the increased prediction accuracy despite the added cost of the finite element models.

Looking more closely at the results, the FEM design is with 38% actually more likely to be changed than the RSM design. However, out of the design changes for the FEM model, a large majority of 34% can be resolved virtually for free by flexibly changing the mold temperature. Only a small fraction of the changes require the adjustment of the wall thickness, and no change requires the increse of the number of tools. Most significant, the marginal part cost has been reduced from $6.25 to $6.04 due to the reduction in wall thickness, subsequently reducing the material cost, the cooling time, and the processing cost. With respect to the improved expected cost, it is advised to invest the effort of obtaining the FEM model. It is important to point out, that after creating the FEM model, the flexible design analysis has to be used to determine the design with the optimal trade-off between the part cost and the change cost. The FEM design shown above is used only to determine the likely effects of the increased prediction accuracy, but cannot be used to determine the ideal design giving the least expected cost.

Table 49: Value of Information Comparison

Design Element	RSM Design	FEM Design
Expected Cost	$6.39	$6.14
Cost of Information	$0.00	$0.20
Combined Cost	$6.39	$6.34
Probability of No Change	83.8%	61.6%
Probability of Any Change	14.8%	38.4%
Probability of Failure	1.3%	0.03%
Probability of Change in Mold Temperature	14.0%	34.1 %
Probability of Change in Thickness	5.9%	3.7%
Probability of Change in Number of Tools	1.8%	0.0%
Probability of Change in Material Type	2.0%	1.8%

7.6 Implementation

The exhaustive analysis of all expected outcomes and possible design changes is computationally expensive. The number of model evaluations increases exponentially with the number of design variables and design responses. In addition, each design change analysis requires an optimization, further increasing the number of required model evaluations. The monitor housing included 4 design variables and 4 design responses, out of which three responses where specified on one side and one response was specified on both sides. This generated 16 possible design changes and 24 possible expected outcomes. Therefore, 384 design changes where investigated and optimized. A design change optimization required about 40 iterations, creating the need for 15,360 model evaluations. The monitor housing example was implemented using Microsoft Excel, and utilizing a built in Solver module for optimization. The calculation of one flexible design analysis required about 30 minutes on a Pentium III computer with 700 MHz. The optimization of the expected cost required 20-30 individual flexible design evaluations, pushing the computation time to 15 hours.

7.7 Summary

The flexible design methodology was demonstrated for the development of an injection-molding monitor housing. The methodology successfully reduced the expected cost by over 13%, representing a value of $470,000, compared to the optimal robust design. This improvement in cost does not include the additional benefits of the reduced development time due to design changes. The flexible design methodology also increased the design flexibility: the initial design required 84% of the design changes via the expensive adjustment of the number of tools and the wall thickness, compared to 43% for the flexible design. Therefore, the flexible design methodology determined a design with an ideal trade-off between the marginal part cost and the cost due to design changes, providing the design with the least expected cost. In addition, the value of information was investigated for a response surface model and a finite element model, indicating that the more accurate FEM model should significantly reduce the expected cost.

CHAPTER 8

CONCLUSION

This chapter summarizes the research described in this dissertation. The new and unique contributions for the engineering design research community are listed. In addition, further possible research to enhance the flexible design methodology is described and additional possible utilizations are indicated.

8.1 Contributions

Although the described research builds on work by other researchers as cited, several contributions have been made within the scope of this dissertation. This section describes the new and unique research for the engineering design community.

8.1.1 Uncertainty Description and Design Requirement

In order to enable the flexible design methodology, the prediction uncertainty had to be described, and the requirements on the prediction uncertainty defined. While probability distributions for uncertainty and yield estimation methods were previously developed, the combination of these two methods to formulate a quality requirement under uncertainty is a new contribution to the research community. This quality requirement facilitates the design evaluation within the flexible design methodology. The contribution is therefore minor compared to the following major contributions.

8.1.2 A Priori Design Defect Evaluation

One of the major contributions of the research presented in this dissertation is the a priori evaluation of the possible design defects and the possible design changes. The implementation of this idea is novel, unique and a significant advancement in the area of design robustness against uncertainty.

8.1.2.1 A Priori Analysis of Expected Outcomes

The flexible design methodology evaluates the possible expected outcomes based on the a priori analysis of the design with respect to the uncertainty. The actual design responses may differ from the predicted design responses, causing an excessive number of defects and violating the quality requirement. The flexible design methodology describes a new approach that determines the different expected outcomes and the likelihood of these expected outcomes occurring based on the prediction uncertainty distribution. The exact nature of the possible expected outcomes is determined, therefore bringing more clarity to the possible actions to resolve the expected outcomes while still in the design development stage.

8.1.2.2 A Priori Design Change Evaluation

Based on the individual expected outcomes, alternative design changes can be evaluated based on the uncertainty distributions. The flexible design methodology analyzes all possible design changes for a given expected outcome a priori and evaluates the probability of a design change being selected based on economic considerations. The expected cost can then be evaluated including design changes, creating an objective including the cost of the part and the cost of the changes. Previous research did not investigate this trade-off, but rather either ignored the concept of design changes altogether, or focused on one design change if a defect occurs. The approach to consider a possible design change in a trade-off with the cost of the design at the development stage is novel and unique and a major contribution to the research community.

8.1.3 Design Flexibility

Another unique contribution to the research community is the idea of design flexibility, where it is possible to adjust for errors by means of one or more easy to change design variables. Although it is common knowledge that a flexible design, i.e. a design which can be changed easily in order to change the design responses if necessary, is beneficial to the design, the actual evaluation of the expected cost with respect to the design flexibility including the likelihood of changing a design variable is a new and unique contribution to the engineering design research community.

8.1.4 Value of Information

Information is crucial for engineering design, and the use of information has been subject to much research. However, the impact of information, i.e. prediction accuracy, on the likelihood of design changes and the overall cost of the design as demonstrated by using the flexible design methodology is a novel approach to measure the value of information content. The simulation of design predictions using an economic model with less prediction accuracy is also a new contribution to the research community. This research allows the design team to compare the likely benefits of different prediction models prior to development of these models. Therefore, the described methodology reduces the product development time and the product development cost by aiding the design team with the selection of a desirable prediction model.

8.2 Future Research

The research presented in this dissertation can be used to facilitate additional research, and selected elements of the methodology can be refined. These possibilities for further research are listed below.

8.2.1 Development Time

The flexible design methodology focuses entirely on the cost of the design, including the cost of possible design changes. However, it is not only the cost of a design that is of significance, but also, the time required to develop the complete system design. The problem in the evaluation of the design development time is to estimate the delays due to unexpected problems and design changes. The flexible design methodology can be expanded to include the time required for design changes in the trade-off analysis. This extension would provide the design team with a tool to determine the delays due to changes for a given design a priori and to select a design accordingly in order to reduce the time for changes and to include the required change time in the design development plan.

8.2.2 Concurrent Design

Within the flexible design methodology, the uncertainty causing a one-time difference between the predicted response and the mean actual response is evaluated only on the basis of the prediction uncertainty. However, there are additional sources of uncertainty besides the prediction models, such as the selection of the design variables or the specifications. During the design development stage, some design variables might not be known at the beginning of the design development but rather are determined during the development process. Therefore, the existence of these design variables is, too, uncertain. For example, a flexible design might be developed during the development phase, where the exact value of one design variable is not yet known, as the variable might be dependent on another design decision at a later stage. This flexible design may be able to compensate for this uncertainty a priori, or it may include an easy to change design variable to adjust the design for the uncertainty, thus improving the design and reducing the overall cost.

A frequent situation where some design variables might not yet be known exactly beforehand is concurrent engineering. One goal of concurrent engineering is to overlap design development tasks in time to reduce the overall development time. This, however, comes at the cost of uncertainty, where a required design variable might not yet be known at the start of a design development task, since the variable depends on a not yet completed previous task. This enables the design team to determine a flexible design with a small change cost and time, improving the concurrent engineering design process. Not only can the design be improved, but also the concurrent engineering tasks can be optimized to reduce the overall development time. If it is possible to determine the time required for a design task including possible changes due to uncertainty, it is also possible to compare different variable uncertainties affecting the design cost and time. This would, in turn enable the design team to determine the ideal order and overlap between concurrent design tasks to reduce the overall cost and time of the design process.

8.2.3 System Design

The flexible design methodology enables the evaluation of designs with a small to medium number of design variables and design responses, as frequently found in component design or simple design systems. Further research has to be invested in order to improve the methodology for complex system design with a large number of design variables.

8.2.4 Change Cost Improvement

The method presented in this dissertation to determine the cost of a design change is on a very basic level. It is possible to improve this method. The change cost depends not only on the changed variables, but also on the direction and the magnitude of the change. For example, it requires much less effort to increase the hole diameter in a tool by drilling a larger hole than to reduce the hole diameter by fitting an insert into the hole. Improvements in change cost estimates will greatly increase the validity of results from the flexible design methodology.

8.2.5 Enhanced Trade-Off between Cost and Quality

At the current stage, the flexible design methodology has strict limits on the quality requirement. If the design has only a slightly smaller quality than required, a design change is initiated. However, in this case, the expense of the design change might exceed the benefit of the slightly improved quality, and economic considerations would advise against a design change. This situation happens very rarely for large number of parts, where the change cost is fixed, but the benefit of an improved quality increases with the number of parts. However, it is possible for low production quantities that the cost of the design change outweighs the benefits of the improved quality. A trade-off between the cost of a design and the quality might be included in the flexible design methodology to improve the prediction accuracy.

8.2.6 Predictive Model Feedback

The flexible design methodology analyzes in detail the possible defects and design changes a priori in the design development stage, to reduce the overall expected cost including the part cost and the change cost. However, no further guidance is provided once the design is built and might have to be changed if a defect occurs. It would be valuable research to provide the design teams with methods to incorporate the information gained by the creation of the single design point into the prediction model to improve the prediction accuracy.

8.3 Summary

The flexible design methodology assesses the possible defects of a design and determines possible design changes a priori, integrating the results into an overall expected cost. The expected cost enables the adjustment of the design to reduce the overall expected cost during the design development stage, including possible design changes.

The author believes that the handling of uncertainties, i.e. one time offsets between the predicted response and the actual response, is a significant area of needed research. While this dissertation does not solve all problems related to the handling of uncertainty, it is a significant contribution to the research community. And, in due time, the research presented in this dissertation may find its way into the industry practice in order to be included in the design development process as is the robust design method today.

APPENDIX A

PROBABILISTIC METHODS

This appendix provides an overview of selected probabilistic methods for the evaluation of response distributions as a function of the variable distributions. (Robinson 1998) also provides and overview of different probabilistic methods for the evaluation of risk and uncertainty.

A.1 Functional Prediction

A.1.1 Functions of One Variable

The distribution of the response can be determined analytically if the response is a function of one variable, whose probabilistic distribution is known. (Papoulis 1991) presents the equations necessary to solve this problem. Assuming the relation between the variable x and the response y is known.

$$y = g(x)$$

Equation 81

Furthermore, assume the probabilistic density function $pdf^{N}(x)$ of the noise of x is also known. In this case, the probabilistic density function $pdf^{N}(y)$ of the variation of the response y is evaluated as shown below:

$$pdf^N(y) = \frac{pdf^N(x_1)}{|g'(x_1)|} + \ldots + \frac{pdf^N(x_n)}{|g'(x_n)|}$$

Equation 82

In this equation, x_n represents the solutions for the inverse of the functional relation in Equation 81 as shown in Equation 83.

$$\{x_1, \ldots x_n\} = g^{-1}(y)$$

Equation 83

For example, assume in the following system the response *y* is inverse related to the normal distributed variable *x* with a mean at 2 and a standard deviation of ½, with no additional unexplained response variation $pdf^N(y)$. A plot of the normal distribution and the functional relation is shown in Figure 41.

$$y = g(x) = \frac{1}{x}$$
$$pdf^N(x) = \frac{1}{2\sqrt{\pi}} e^{-2(x-2)^2}$$

Equation 84

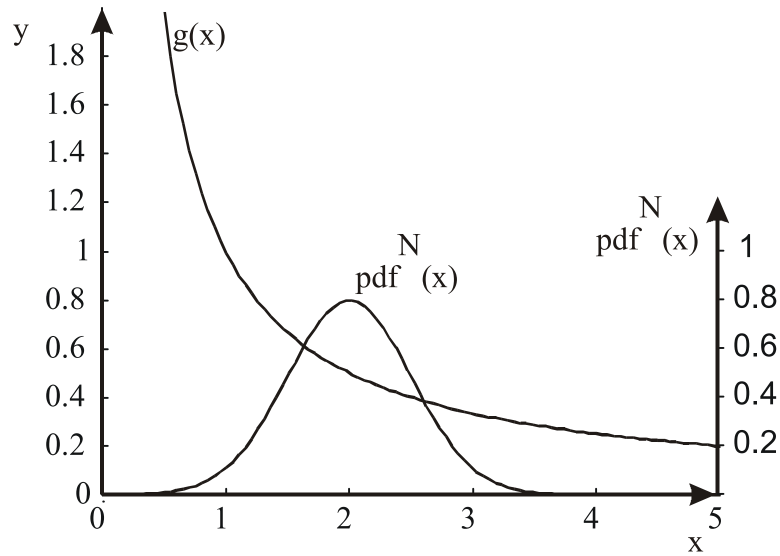


Figure 41: Functions of One Variable

Applying the interim results shown in Equation 85 to Equation 82 gives the probability density function $pdf^N(y)$ of the response y shown in Equation 86. A plot of this response distribution is also shown in Figure 42.

$$g'(x) = \frac{1}{x^2}$$
$$g^{-1}(y) = \frac{1}{y}$$

Equation 85

$$pdf^N(y) = \frac{e^{-2\left(\frac{1}{y}-2\right)^2}\sqrt{\frac{2}{\pi}}}{|y|^2}$$

Equation 86

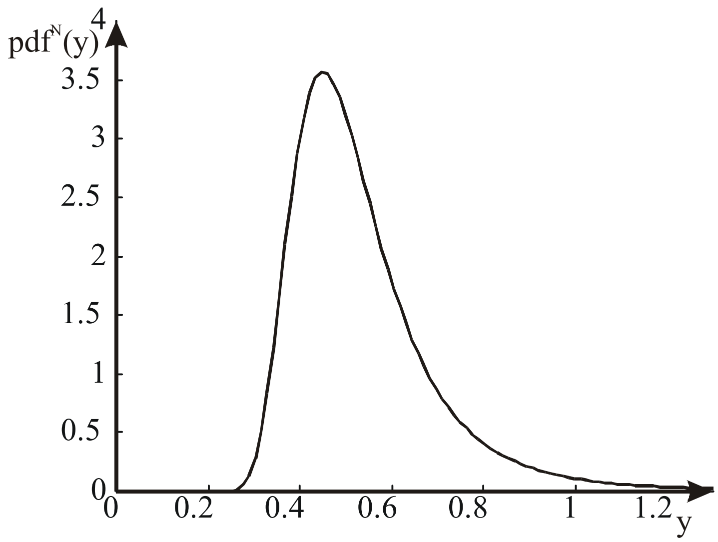


Figure 42: Probability Density Function of the Function of One Variable

The analytical approach gives the exact probability density function for a function of one variable. Unfortunately, this approach includes significant mathematical calculations, and - depending on the complexity of the functional relationships - may not be feasible to solve analytically. Even with the use of numerical methods, these problems are frequently beyond the capabilities of a numerical solver. In addition, there are few engineering design problems, which are only a function of one variable. Rather, a number of different variables are involved.

A.1.2 Functions of Two Variables

A similar approach using a functional evaluation can be done for a function of two variables as shown in Equation 87. In addition to the functional relationship between the response *y* and the variables x_i, the joint probabilistic density functions of $pdf^N(x_1)$ and $pdf^N(x_2)$ of the design variables also have to be known. In this case, the probabilistic density function $pdf^N(y)$ of the response *y* is evaluated as shown in Equation 88.

$$y = g(x_1, x_2)$$

Equation 87

$$pdf^N(y)dy = \iint_{D_z} pdf^N(x_1, x_2)dx_1 dx_2$$

Equation 88

In this equation, D_z represents the area over which the probability is evaluated. This method is demonstrated using an example from (Devore 1995), extending it towards a more general case. The example analyzes a three-component mixture, where the combined weight of two components together is a function of the random distributed weight of two components, with the third component being the remainder towards a fixed total weight. The relation between the response y, representing the combined weight of the two components x_1 and x_2, and the two separate component weights x_1 and x_2 is described below in Equation 89, followed by the joint probability density function $pdf^N(x_1, x_2)$ for x_1 and x_2 in Equation 90.

$$\begin{aligned} &y = x_1 + x_2 \\ &0 \le x_1 \le 1 \\ &0 \le x_2 \le 1 \\ &0 \le y \le 1 \end{aligned}$$

Equation 89

$$pdf^N(x_1, x_2) = 24 x_1 x_2$$

Equation 90

If the component x_1 has a certain amount of the total weight of y, then following Equation 89 the other component x_2 has to have the remaining weight towards y. Hence, it is possible to evaluate the cumulative density function $cdf^N(y)$ and subsequently the probability density function $pdf^N(y)$ for y as shown in Equation 91 and plotted in Figure 43.

$$cdf^N(y) = \int_0^y \int_0^{y-x_2} 24 x_1 x_2 dx_1 dx_2 = y^4$$
$$pdf^N(y) = 4y^3$$

Equation 91

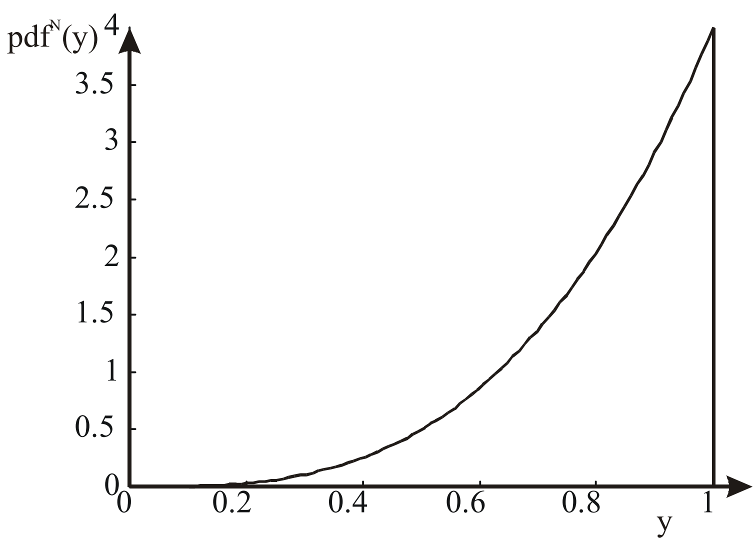


Figure 43: Response Probability Density Function

The separate probability density functions $pdf^N(x_1)$ and $pdf^N(x_2)$ of the variables can be evaluated by integrating the joint probability over the allowed range as shown in Equation 92. Due to symmetry reasons, these two functions are identical for this case, and the function is plotted in Figure 44.

$$pdf^{N}(x_1) = \int_{0}^{1-x_1} 24 x_1 x_2 dx_2 = 12 x_1 \left(1 - x_1^2\right)$$
$$pdf^{N}(x_2) = 12 x_2 \left(1 - x_2^2\right)$$

Equation 92

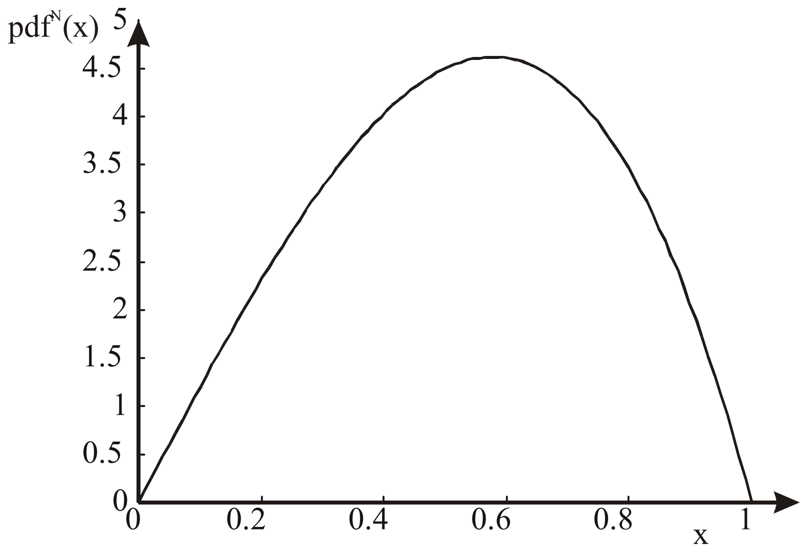


Figure 44: Variable Probability Density Function

The functional evaluation of the response as a function of two variables shares the same advantages and disadvantages as the functional evaluation of the response as a function of one variable. Although the approach is mathematically exact, it cannot model relationships for which the transfer functions are unknown. Furthermore, if the transfer function for variables or noise is estimated, then the imprecise basis does not justify the precise estimation. In addition, the mathematical calculations are very sophisticated, and for relations that are more complex may not always create a solution. Finally, there are very few design relations, which are a function of only two variables. For more detail regarding the mathematical background please refer to (Papoulis 1991).

A.1.3 Functions of More than Two Variables

Engineering design applications frequently have more than two variables, which restricts the methodologies presented above. However, there are two possible ways to analytically evaluate functions of more than two variables. If the variable combinations are independent, then it is possible to split a transfer function into a number of sub functions with only two variables. The probability distribution of the interim response is evaluated, which is then used as input for the overlaying functional relation. By breaking a large transfer function down into a number of sub functions with only two variables, the analytical method for two variables can be applied.

If the transfer function does not allow the use of sub functions or the variables are dependent, the method of evaluating a response as a function of two variables has to be expanded, and multiple integrals have to be used. The underlying equation for a function of three variables is shown in Equation 93. The advantages and disadvantages for an analytical evaluation of a function of three or more variables are identical with the advantages and disadvantages of an analytical evaluation of a function of two variables. Note, however, that the mathematical relations become more complex as more variables are added.

$$pdf^{N}(y)dy = \iiint_{D_z} pdf^{N}(x_1, x_2, x_3)dx_1 dx_2 dx_3$$

Equation 93

A.2 Moment Matching

The moment matching method, also known as error propagation, uses a Taylor series expansion to predict the mean and variance of a function of independent normally distributed random variables. The moment matching method is named due to the use of the first moment - the mean - and the second moment - the variance - of the probability distributions. Assume the following system shown in Equation 94 where the response *y* is a function of normal distributed design variables x_i with mean μ_{x_i} and deviation σ_{x_i}.

$$y = g(x_1, x_2, \ldots x_n)$$
$$pdf^N(x_i) = \frac{1}{\sqrt{2\pi}\sigma_{x_i}} e^{\frac{-(x_i-\mu_i)^2}{2\sigma_{x_i}^2}}$$

Equation 94

In this case, the expected mean of the response μ_y is a direct function of the expected means of the design variables μ_{x_i}. Depending on the functional relation between the design variables x_i and the design response *y*, the response distribution might not necessarily be a standard distribution. However, this method assumes a normal distributed response. The evaluation of the response mean and deviation uses the derivatives of the functional relations as shown in Equation 95.

$$\mu_y = g(\mu_{x_1}, \mu_{x_2}, \ldots \mu_{x_n})$$
$$\sigma_y = \sqrt{\sum_{i=1}^{n} \left(\frac{dg(x_1, x_2, \ldots, x_n)}{dx_i} \sigma_{x_i} \right)^2}$$

Equation 95

The advantage of this method is the ease of computation, yet the moment matching method is limited as the design variables are assumed to be independent and normal distributed, creating a normal distributed response. This may reduce the accuracy of the prediction if the involved design variables are not normally distributed. (Parkinson et al. 1993) for example uses a moment matching method to develop a general robust design approach similar to the approach presented within this dissertation.

A.3 Monte Carlo

The Monte Carlo method evaluates a function of one or more random variables by sampling response values and making conclusions for the distribution of the function response. Using randomly distributed design variables, a design is evaluated repeatedly generating slightly different design responses due to the random nature of the design variables. Based on the set of design response values for a given design, the yield can be either estimated by counting the number of feasible parts, or by fitting a response distribution through the sample and integrating over this response distribution.

This method is best demonstrated using the example of section A.1.2 assuming a normal distributed input variable with a mean of two and a standard deviation of ½ and an inverse relationship between the variable and the response as shown in Equation 84. For 3,000 sample trials, the histogram of the input variable was evaluated as shown in Figure 45. This figure can be compared directly to he functional evaluation as shown in Figure 41, displaying a very similar curve shape.

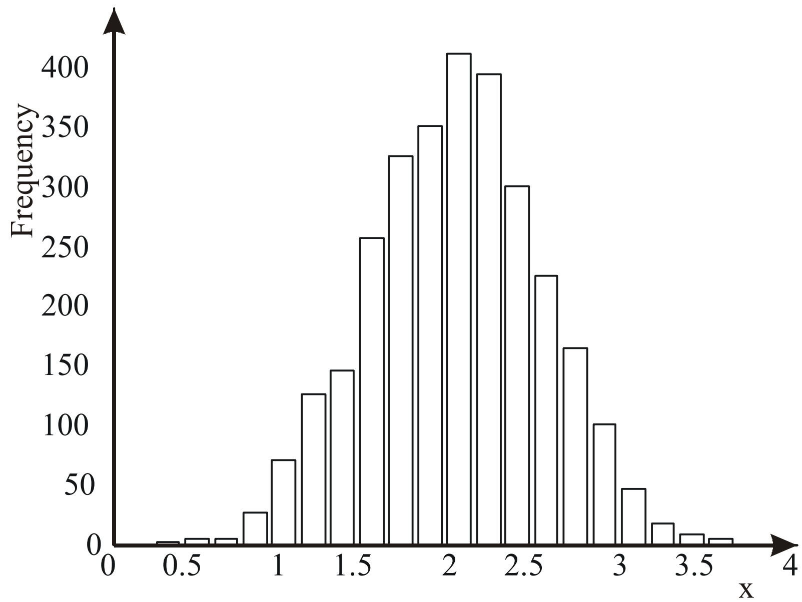


Figure 45: Input Sample Values

Applying sample values to the functional relationship, 3,000 response values were evaluated. The histogram of the responses is shown in Figure 46. Again, this histogram shows an approximately similar curve as the functional evaluation shown in Figure 42. Note the very long tail on the right hand side exists due to some samples being close to zero, which drives the inverse of the variable, i.e. the response, close to infinity.

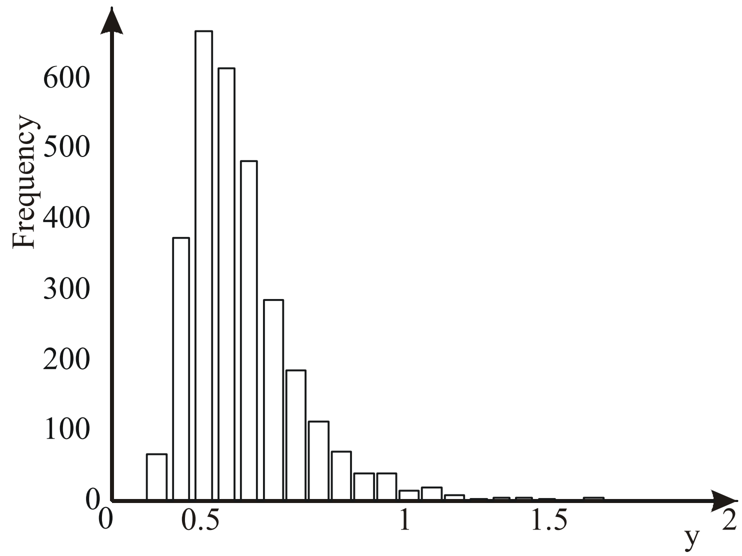


Figure 46: Response Sample Values

Small differences between the histograms above and the functional evaluation described earlier are due to the random nature of the Monte Carlo analysis. This analysis has a number of advantages and disadvantages. One advantage is that this method can also be used on processes for which a functional relation is not known, and the responses can be determined only by using experiments. In this case, a small number of experiments are performed, generating slightly different responses due to natural variation in the system. These responses can now be used to fit a probability density function through these responses, generating an estimate of the response distribution. This method is also used in a similar way to determine the actual response distributions during production, again measuring the design responses of multiple products and fitting a probability distribution through the sample.

Another advantage is the ease of use. While other yield evaluations require a significant knowledge of mathematics, Monte Carlo analyses are easy to perform and analyze, making this method very popular in industry.

On the downside, however, there is a significant disadvantage. In order to predict the distribution of the design responses with a reasonable accuracy, a significant number of evaluations have to be performed. The computation time required to evaluate a Monte Carlo analysis depends on the complexity of the underlying simulations and the number of samples evaluated. Unfortunately, even for a simulation with a reasonable computation time, the large number of sample points needed would significantly increase the total computation time.

Fortunately, there are some ways to reduce the number of runs. Assumptions are frequently made regarding the design responses, fitting a standard probabilistic distribution onto the response data samples. Most frequently, the mean and standard deviation of the samples are evaluated and a standard normal distribution is assumed. If other standard distributions besides the normal distribution are expected, it is possible to fit a variety of probability density distributions on the data, and to evaluate the goodness of fit, selecting the distribution with the closest match to the data sample.(Suresh 1997) describes different Monte Carlo analysis techniques in more detail.

A.4 Comparison of the Presented Methods

The above methods all have advantages and disadvantages. Depending on the application it has to be decided which method is best. Table 50 compares some of the advantages and disadvantages of the above methods. If normal distributions can be assumed without too much loss of accuracy, then the moment matching method provides the largest advantage. However, if non-normal distributed variables or responses have to be considered, then the Monte Carlo method would be preferred except for the simplest models for which a functional prediction can be evaluated. For similar comparisons between different statistical methods, please refer to (Du and Chen 1999; Koch and Mavris 1998).

Table 50: Comparison of Response Distribution Prediction Methods

	Functional Prediction	Moment Matching	Monte Carlo
Uses General Probability Distribution	Yes	Normal Distribution Only	Yes
Predict General Probability Distribution	Yes	Normal Distribution Only	Yes
Can use Simulations or Experiments	No	No	Yes
Functional Relation Required	Yes	Yes	No
Accuracy	Exact	Medium	Depends on sample size
Mathematical Effort	Extreme, may not be tenable at all	Medium	Low
Computation Time	Medium	Low	High

APPENDIX B

I-BEAM EXAMPLE

This is a summary of the equations used for the I-Beam example. The example was realized using Mathematica Version 4 from Wolfram Research. The Mathematica file is shown below.

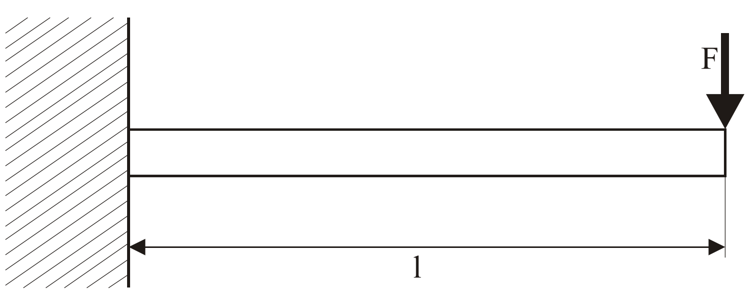
I-Beam Deflection Example

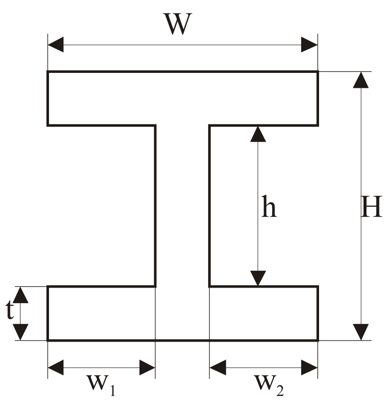
Overview

This document is grouped in several subsections. First, the analytical model will be described and derived. Second, based on the analytical model a quadratic design of experiments will be used to fit a response surface on the data pointscalculated from the analytical model. Third, the RSM model will be verified and compared to the Analytical model. Fourth, a flexible design method will be performed. Fifth, different designs are compared using the flexible design methodology.

Physical Example

This example describes the flexible design evaluation for an I-beam with a fixed support and a point load at the other end. There are two main variables: the beam height *H* and the modulus *EMod* of the material. Both are considered to be continuous. The deflection is the specified response, where the deflection has to be less than the upper specification limit. The marginal part cost is also evaluated. The pictures below show the cross section and the side view of the beam, including the nomenclature of the dimensions.





Initialization and Functional Relations of the Analytical Models

Setup of the Known Values and Specifications

Clears up the variable space and loads required packages.

```
In[517]:= ClearAll["Global`*"];
          << Statistics`NormalDistribution`
          << Graphics`Legend`
```

Defines the Upper and Lower Constraint Limits of the Design Variables, which form the design space. The height *H* varies from the lower limit *HL* to the upper limit *HH* and the E modulus *EMod* varies form the lower limit *EL* to the upper limit *EH*.

```
In[520]:= HL = 30; HH = 60; EL = 90000; EH = 185000;
```

This defines the Upper Specification Limit USL in mm. The beam fails if the deflection is above this limit.

```
In[521]:= USL = 3;
```

Quality Requirement

The quality Requirement Alpha describes the required number of deviations distance between the mean response and the specification limit in order to ensure consistent quality. An Alpha of three represents 99.87% good parts for a one sided specification and 99.74% for a two sided specification.

```
In[522]:= Alpha = 3;
```

Functional Relations

This section describes the functional relations of the I-Beam example. The equations can be found in most engineering manuals. The following equation determined the Area Integral of I Beam, with *W, H, w, h* as described in the pictures above.

In[522]:= $Iy = \frac{1}{12}\left(-h^3\, w + H^3\, W\right);$

This represents the deflection of the beam, where *F* is the load in N and *l* is the length in mm

```
In[524]:= DeflAnal = Simplify[F * l ^ 3 / (3 * EMod * Iy)]
```

Out[524]= $\frac{4\, F\, l^3}{-EMod\, h^3\, w + EMod\, H^3\, W}$

Calculation of the vVolume of the beam.

```
In[525]:= V = l (-h w + H W);
```

Marginal part cost of the beam, where d is the density in kg/mm^3 and MC is the material cost in $/kg.

```
In[526]:= CostAnal = Simplify[V * d * MC]

Out[526]= d l MC (-h w + H W)
```

Assumptions

The following assumptions have been made:

Constant Wall Thickness *t* (mm)

Constant Beam Width *W* (mm)

Constant Force Load *F* (N)

Constant Beam Length *l* (mm)

Constant Density *d* (Kg/mm^2)

Noise Standard Deviation of the Height *HN* (mm)

Noise Standard Deviation of the modulus *EModN* (N/mm^2)

The noise distribution are assumed to be standard normal distributed with a mean of zero and a standard deviation as shown below.

```
In[527]:= t = 5;
          W = 30;
          F = 100;
          l = 1000;
          d = 7200 / (1000 ^ 3);
          HN = 1 / 10;
          EModN = 1700;
```

Simplifications

The relation of the beam web height *h* with the height *H* and the thickness *t* is simplified. The relation of the web with *w* based on the beam width *W* and the thickness *t* is shown. Material Cost *MC* ($/kg) is a linear function between $500/ton and $540/ton for the low and high modulus.

```
In[534]:= h = H - 2 * t;
          w = W - t;
          LoCost = 500; HiCost = 540;
          MC = Simplify[
            (LoCost + (HiCost - LoCost) * (EMod - EL) / (EH - EL)) / 1
```

Out[537]= $\frac{1097500 + \text{EMod}}{2375000}$

Summary

Deflection Equation as a function of the height *H* and the modulus *EMod*

```
In[538]:= DeflAnal = Simplify[DeflAnal]
```

Out[538]= $\frac{80000000000}{\text{EMod}\,(-5\,(-10 + H)^3 + 6\,H^3)}$

Deflection Noise Standard Deviation as a function of the height *H* and the modulus *EMod*. The noise distributio of the deflection is assumed to be normal distributed.

```
In[539]:= DeflNAnal = Simplify[√((D[DeflAnal, H] * HN) ^ 2 +
            (D[DeflAnal, EMod] * EModN) ^ 2)]
```

$$\text{Out[539]=}\ 8000000000\sqrt{\left(\left(9\,EMod^2\,(-500+100\,H+H^2)^2+289000000\,(5000-1500\,H+150\,H^2+H^3)^2\right)/\left(EMod^4\,(5000-1500\,H+150\,H^2+H^3)^4\right)\right)}$$

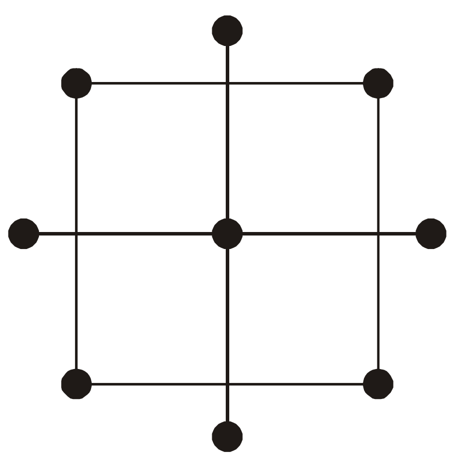
Cost equations a function of the height *H* and the modulus *EMod*.

```
In[540]:= CostAnal = Simplify[CostAnal]
```

$$\text{Out[540]=}\ \frac{9\,(1097500+EMod)\,(50+H)}{593750000}$$

Design of Experiments

The following section describes the creation of a CCD Design of Experiments to fit a quadratic response surface to the design space. The figure below shows the general layout of the design of experiments



Data Evaluation

The CCD Design of Experiments has nine data points, four at the four corners of the design space, one in the center and four +- $\sqrt{2}$ away from the center on the center axes.

Below is the set up of the complete variable matrix including the constant parts and the interactions. The rows represent the different data points. The first column is the constant value. The second column is the beam height, the third is the modulus, the fourth is the height times the modulus, the fifth is the height squared, and the sixth is the modulus squared.

```
In[541]:= HMatrix = Transpose[{{HL, HL, HH, HH, (HL + HH) / 2,
            (HL + HH) / 2, 1/2 (HL + √2 HL + HH - √2 HH),
            1/2 (HL - √2 HL + HH + √2 HH), (HL + HH) / 2}}] // N;
          EMatrix =
           Transpose[{{EL, EH, EL, EH, 1/2 (EL + √2 EL + EH - √2 E
            1/2 (EL - √2 EL + EH + √2 EH),
            (EL + EH) / 2, (EL + EH) / 2, (EL + EH) / 2}}] // N;
          ConstMatrix = Table[1, {i, 9}, {j, 1}];
          AllVar  = Transpose[{Transpose[ConstMatrix][[1]],
             Transpose[HMatrix][[1]], Transpose[EMatrix][[1]],
             Transpose[HMatrix * EMatrix][[1]],
             Transpose[HMatrix^2][[1]],
             Transpose[EMatrix^2][[1]]}];
          SetPrecision[MatrixForm[AllVar], 4]
```

Out[545]//MatrixForm=

1.000	30.00	9.00×10^{4}	2.700×10^{6}	900.	8.10 ×
1.000	30.00	1.850×10^{5}	5.55×10^{6}	900.	3.42 ×
1.000	60.0	9.00×10^{4}	5.40×10^{6}	3.60×10^{3}	8.10 ×
1.000	60.0	1.850×10^{5}	1.110×10^{7}	3.60×10^{3}	3.42 ×
1.000	45.0	7.03×10^{4}	3.16×10^{6}	2025.	4.95 ×
1.000	45.0	2.047×10^{5}	9.21×10^{6}	2025.	4.19 ×
1.000	23.79	1.375×10^{5}	3.27×10^{6}	566.	1.891 ›
1.000	66.2	1.375×10^{5}	9.10×10^{6}	4.38×10^{3}	1.891 ›
1.000	45.0	1.375×10^{5}	6.19×10^{6}	2025.	1.891 ›

This is the calculation of the responses using the analytical model. The rows refer to the nine data points as described above. The first column shows the deflection. The second column shows the standard deviation of the deflection due to the noise. The third column shows the marginal part cost.

In[546]:=
```
H = HMatrix; EMod = EMatrix;
AllRes = Transpose[
  {Transpose[DeflAnal][[1]], Transpose[DeflNAnal][[1
   Transpose[CostAnal][[1]]}];
SetPrecision[MatrixForm[AllRes], 4]
Clear[H, EMod];
```

Out[548]//MatrixForm=

7.29	0.1505	1.440
3.54	0.0440	1.555
1.325	0.02560	1.980
0.644	0.00648	2.138
3.42	0.0848	1.682
1.176	0.01167	1.875
8.60	0.1414	1.381
0.682	0.00879	2.176
1.750	0.02364	1.778

This section shows the creation of the prediction models by solving the equation described by the data matrices above. Note, that the response surface model of the standard deviation may predict a standard deviation of equal or less than zero for certain design points. To avoid complications, this response surface model is limited to values above the smallest standard deviation from the data points. The resulting models are shown below.

```
In[550]:= Model = Inverse[Transpose[AllVar] . AllVar] .
            Transpose[AllVar] . AllRes;
          DeflMod  = Model[[1, 1]] + Model[[2, 1]] * H +
            Model[[3, 1]] * EMod + Model[[4, 1]] * H * EMod +
            Model[[5, 1]] * H^2 + Model[[6, 1]] * EMod^2;
          DeflNMod = Model[[1, 2]] + Model[[2, 2]] * H +
            Model[[3, 2]] * EMod + Model[[4, 2]] * H * EMod +
            Model[[5, 2]] * H^2 + Model[[6, 2]] * EMod^2;
          DeflNMod = Max[Min[AllRes[[All, 2]]], DeflNMod] // N;
          CostMod  = Model[[1, 3]] + Model[[2, 3]] * H +
            Model[[3, 3]] * EMod + Model[[4, 3]] * H * EMod +
            Model[[5, 3]] * H^2 + Model[[6, 3]] * EMod^2;
```

The following section can be used to determine the goodness of fit of the model.

```
In[555]:= << Statistics`NonlinearFit`
          Clear[BA, BB, BC, BD, BE, BF, XA, XB, x1, x2];
          RegEq = BA + BB x1 + BC x2 + BD x1 * x2 + BE * x1^2 + BF * x2^2
          vars = {x1, x2};
          pars = {BA, BB, BC, BD, BE, BF};
          data = Transpose[{HMatrix[[All, 1]],
             EMatrix[[All, 1]], AllRes[[All, 1]]}];
          NonlinearRegress[data, RegEq, vars, pars];
          data = Transpose[{HMatrix[[All, 1]],
             EMatrix[[All, 1]], AllRes[[All, 2]]}];
          NonlinearRegress[data, RegEq, vars, pars];
          data = Transpose[{HMatrix[[All, 1]],
             EMatrix[[All, 1]], AllRes[[All, 3]]}];
          NonlinearRegress[data, RegEq, vars, pars];
```

This is the response surface equation for the deflection .

```
In[566]:= DeflMod
```

```
Out[566]= 32.8011 - 0.000093542 EMod + 9.16745 × 10^-11 EMod^2 -
          0.865935 H + 1.0741 × 10^-6 EMod H + 0.0061231 H^2
```

This is the response surface equation for the deflection noise standard deviation.

```
In[567]:= DeflNMod

Out[567]= Max[0.00647656,
           0.74022 - 3.32778 × 10^-6 EMod + 4.8944 × 10^-12 EMod^2 -
            0.0169214 H + 3.06483 × 10^-8 EMod H + 0.000108782 H^2]
```

This is the response surface equation for the cost.

```
In[568]:= CostMod

Out[568]= 0.831789 + 7.57895 × 10^-7 EMod + 7.80652 × 10^-24 EMod^2 +
          0.0166358 H + 1.51579 × 10^-8 EMod H - 1.12757 × 10^-17 H^2
```

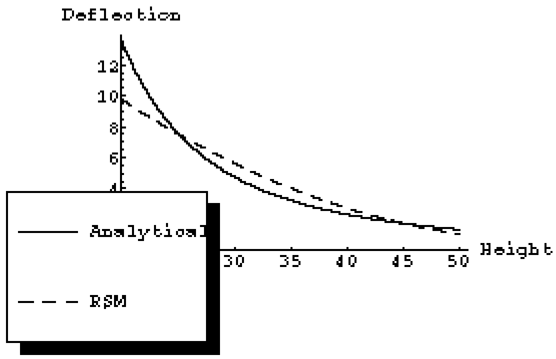
Model Validation

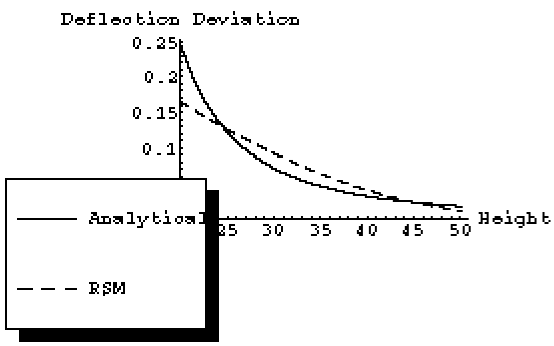
This section validates the Response Surface Model by using the analytical model

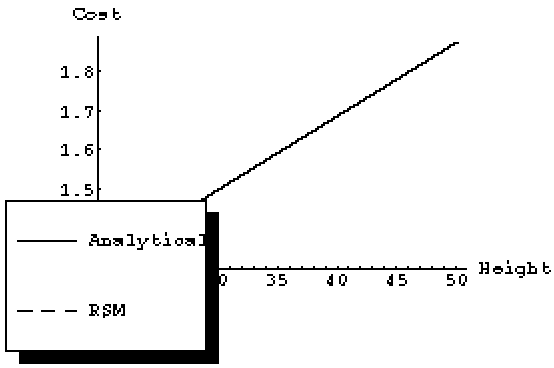
Data Evaluation

Compares the values by varying the heigh *H*t, while the modulu *EMod*s is at the center. The straight line is the functional model, the dashed line is the response surface model.

```
In[569]:= ClearAll[H, EMod];
          EMod = (EH + EL) / 2;
          Plot[{DeflAnal, DeflMod}, {H, 20, 50},
           PlotStyle → {GrayLevel[0], Dashing[{.03}]},
           AxesLabel → {"Height", "Deflection"},
           PlotLegend → {"Analytical", "RSM"}];
          Plot[{DeflNAnal, DeflNMod}, {H, 20, 50},
           PlotStyle → {GrayLevel[0], Dashing[{.03}]},
           AxesLabel → {"Height", "Deflection Deviation"},
           PlotLegend → {"Analytical", "RSM"}];
          Plot[{CostAnal, CostMod}, {H, 20, 50},
           PlotStyle → {GrayLevel[0], Dashing[{.03}]},
           AxesLabel → {"Height", "Cost"},
           PlotLegend → {"Analytical", "RSM"}];
          ClearAll[H, EMod];
```

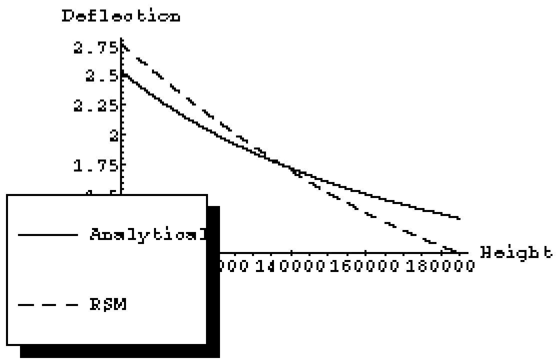

Deflection
12
10
8
6
Analytical
RSM
30
35
40
45
50
Height

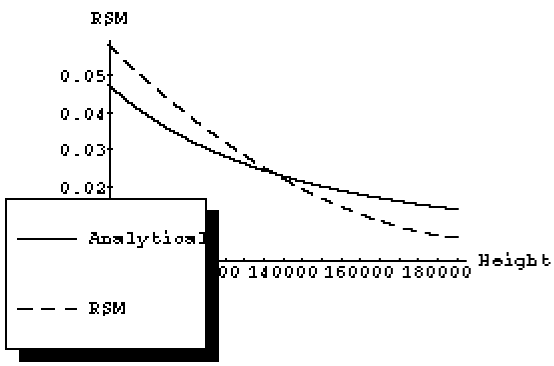

Deflection Deviation
0.25
0.2
0.15
0.1
Analytical
RSM
25
30
35
40
45
50
Height

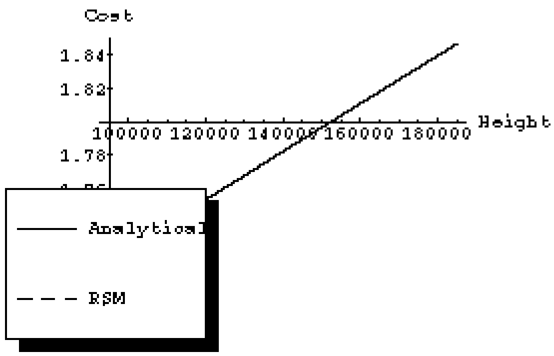


Compares the values by varying the modulus *EMod*, while the height *H* is at the center. The straight line is the functional model, the dashed line is the response surface model.

```
In[575]:= ClearAll[H, EMod];
          H = (HH + HL) / 2;
          Plot[{DeflAnal, DeflMod}, {EMod, 95000, 185000},
           PlotStyle → {GrayLevel[0], Dashing[{.03}]},
           AxesLabel → {"Height", "Deflection"},
           PlotLegend → {"Analytical", "RSM"}];
          Plot[{DeflNAnal, DeflNMod}, {EMod, 95000, 185000},
           PlotStyle → {GrayLevel[0], Dashing[{.03}]},
           AxesLabel → {"Height", "RSM"},
           PlotLegend → {"Analytical", "RSM"}];
          Plot[{CostAnal, CostMod}, {EMod, 95000, 185000},
           PlotStyle → {GrayLevel[0], Dashing[{.03}]},
           AxesLabel → {"Height", "Cost"},
           PlotLegend → {"Analytical", "RSM"}];
          ClearAll[H, EMod];
```


Deflection
2.75
2.5
2.25
2
1.75
Analytical
RSM
000 140000 160000 180000
Height


RSM
0.05
0.04
0.03
0.02
Analytical
RSM
00 140000 160000 180000
Height



Mean and Standard Deviation of Prediction Error

Calculates the Mean of the Error due to the fitting of the response surface model. These are the TRUE VALUES, as the functions are integrated for the whole design space. Note, that these values are NOT used for the later flexibility evaluation, as the values later are the result of 10 random sample points within the design space. The mean error is caluclated below.

```
In[580]:= ClearAll[H, EMod];
          ErrorFun = DeflMod - DeflAnal;
          Prob = 1 / ((HH - HL) * (EH - EL));
          ProbOfPoint = If[H > HL, If[H < HH,
             If[EMod > EL, If[EMod < EH, Prob, 0], 0], 0], 0];
          MeanError =
           NIntegrate[ErrorFun * Prob, {H, HL, HH}, {EMod, EL, EH

Out[584]= 0.118416
```

Calculates the standard deviation of the error for the whole design Space. TRUE VALUES! Again, the deviation used for the flexible design analysis is different and based on 10 random sample points.

```
In[585]:= ClearAll[H, EMod];
          ErrorDeviation =
           Sqrt[NIntegrate[(ErrorFun - MeanError)^2 * Prob,
             {H, HL, HH}, {EMod, EL, EH}]]

Out[586]= 0.452064
```

Values of Selected Points

Calculates selected data points. One point is shown per row. The columns are labeled.

```
In[587]:= ClearAll[H, EMod];
          VIndex = {{"H", "EMod", "DeflAnal", "DeflNAnal",
             "CostAnal", "DeflMod", "DeflNMod", "CostMod" }};
          VEq = {{    H,    EMod,   DeflAnal,    DeflNAnal,
             CostAnal,   DeflMod,   DeflNMod,    CostMod}};
          EMod = EL; H = HL; VA = VEq // N;
          EMod = EH; H = HL; VB = VEq // N;
          EMod = EL; H = HH; VC = VEq // N;
          EMod = EH; H = HH; VD = VEq // N;
          EMod = (EH + EL) / 2; H = (HH + HL) / 2; VE = VEq // N;
          EMod = (EH + EL) / 2; H = HL; VF = VEq // N;
          EMod = (EH + EL) / 2; H = HH; VG = VEq // N;
          EMod = EL; H = (HH + HL) / 2; VH = VEq // N;
          EMod = EH; H = (HH + HL) / 2; VI = VEq // N;
          EMod = EL; H = (HH + HL) / 2; VJ = VEq // N;
          EMod = 182150; H = 34.8; VK = VEq // N;
          VM = {VIndex[[1]], VA[[1]],
            VB[[1]], VC[[1]], VD[[1]], VE[[1]], VF[[1]],
            VG[[1]], VH[[1]], VI[[1]], VJ[[1]], VK[[1]]};
          MatrixForm[SetPrecision[VM, 3]]
          ClearAll[H, EMod];
```

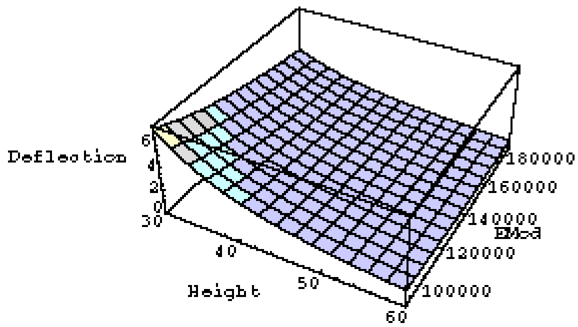
Out[602]//MatrixForm=

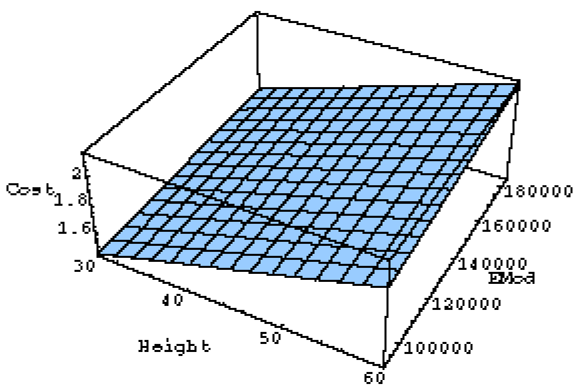
H	EMod	DeflAnal	DeflNAnal	CostAnal	DeflMo
30.0	9.0×10^4	7.3	0.151	1.44	7.6
30.0	1.85×10^5	3.5	0.044	1.56	4.1
60.	9.0×10^4	1.32	0.0256	1.98	1.01
60.	1.85×10^5	0.64	0.0065	2.14	0.64
45.	1.38×10^5	1.75	0.0236	1.78	1.75
30.0	1.38×10^5	4.8	0.071	1.50	5.6
60.	1.38×10^5	0.87	0.0113	2.06	0.62
45.	9.0×10^4	2.67	0.053	1.71	2.91
45.	1.85×10^5	1.30	0.0139	1.85	1.01
45.	9.0×10^4	2.67	0.053	1.71	2.91
35.	1.82×10^5	2.49	0.0292	1.64	2.89

Response Surfaces

Plots different 3D graphs of the response surfaces for the analytical model.

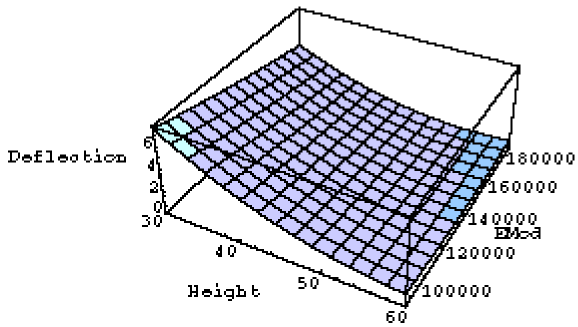
```
In[604]:= ClearAll[H, EMod];
Plot3D[DeflAnal, {H, HL, HH}, {EMod, EL, EH},
 AxesLabel → {"Height", "EMod", "Deflection"}];
Plot3D[CostAnal, {H, HL, HH}, {EMod, EL, EH},
 AxesLabel → {"Height", "EMod", "Cost"}];
ClearAll[H, EMod];
```

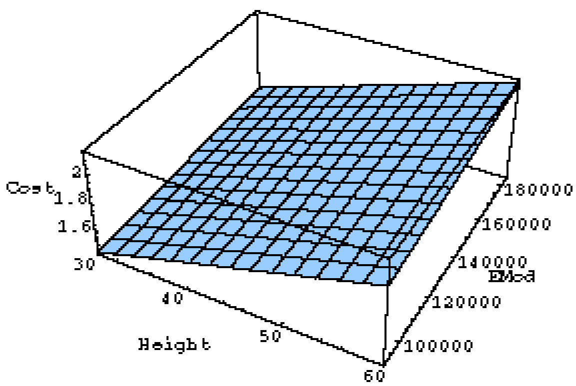




Plots different 3D graphs of the response surfaces for the Response Surface Model

```
In[608]:= ClearAll[H, EMod];
          Plot3D[DeflMod, {H, HL, HH}, {EMod, EL, EH},
           AxesLabel → {"Height", "EMod", "Deflection"}];
          Plot3D[CostMod, {H, HL, HH}, {EMod, EL, EH},
           AxesLabel → {"Height", "EMod", "Cost"}];
          ClearAll[H, EMod];
```





Exhaustive Search for Minimum Part Cost

Optimizes the models.It is possible to use either the analytical models (*DeflAnal, DeflNAnal, CostAnal*) or the RSM models (*DeflMod, DeflNMod, CostMod*). The selection of the models is decided below by changing teh order of the next two selections.

```
In[612]:= Defl = DeflAnal;
          DeflN = DeflNAnal;
          Cost = CostAnal;

In[615]:= Defl = DeflMod;
          DeflN = DeflNMod;
          Cost = CostMod;
```

Determines the minimum cost for the given model while satisfying the quality requirement. The *CostTable* represents the cost for a *Resolution* by *Resolution* matrix, where the *H* increases with the rows and the *EMod* increases with the Columns. The table is not shown here, as it is a 100 by 100 matrix.

```
In[618]:= ClearAll[H, EMod];
          Resolution = 101; Penalty = 5;
          CostTable = Table[
             Cost + (Sign[(Defl + Alpha * DeflN) - USL] + 1) * Penalt
             {H, HL, HH, ((HH - HL) / (Resolution - 1))},
             {EMod, EL, EH, ((EH - EL) / (Resolution - 1))}] // N;
          (* Left to right is increasing EMod,
           Top to bottom is increasing H *)
          MinCostPos = Position[CostTable, Min[CostTable]];
          Hu = HL + ((HH - HL) / (Resolution - 1) *
               (MinCostPos[[1, 1]] - 1)) // N;
          Eu = EL + ((EH - EL) / (Resolution - 1) *
               (MinCostPos[[1, 2]] - 1)) // N;
          MinCostResults = Table[0, {a, 3}, {b, 2}];
          MinCostResults[[1, 1]] = "H";
          MinCostResults[[2, 1]] = "EMod";
          MinCostResults[[3, 1]] = "Cost";
          MinCostResults[[1, 2]] = Hu;
          MinCostResults[[2, 2]] = Eu;
          MinCostResults[[3, 2]] = Min[CostTable];
          MatrixForm[SetPrecision[MinCostResults, 5]]
          ClearAll[H, EMod];

Out[631]//MatrixForm=
```

$$\begin{pmatrix} \text{H} & 34.80 \\ \text{EMod} & 1.8215 \times 10^5 \\ \text{Cost} & 1.6448 \end{pmatrix}$$

Flexible Design Methodology

Analyzes the flexible design methodology for a given initial design.

Initial Design & Setup

It is possible e to use either the analytical models (*DeflAnal, DeflNAnal, CostAnal*) or the RSM models (*DeflMod, DeflNMod, CostMod*). The selection of the models is decided below by swapping between the two different definition cells shown below.

```
In[633]:= Defl = DeflAnal;
          DeflN = DeflNAnal;
          Cost = CostAnal;
```

```
In[636]:= Defl = DeflMod;
          DeflN = DeflNMod;
          Cost = CostMod;
```

This defines the mean and standard deviation of the uncertainty distribution for the deflection. Two sources of uncertainty can be used. The *MeanUAnal* and *DevUAnal* one is to represent the uncertainty of the analytical model, where for example holes for attachments or rounded corners can weaken the beam, whereas a welded attachment may strengthen the beam. As these effects are not included in the analytical model, there might be some prediction errors of the analytical model. The *MeanURSM* and the *DevURSM* represents the uncertainty of the response surface model due to fitting errors. Note, that these values differ from the absolute mean and standard deviation of the RSM model as calculated above, because the values below are derived from 10 random sample points.

If the analytical model is used, only the analytical uncertainty is used. If the RSM model is used, both the analytical uncertainty and the model fitting uncertainty are used. To switch between the uncertainty models, please move the appropriate sections to the end of this statement.

```
In[639]:= MeanUAnal = 0; DevUAnal  = 0.05;
          MeanURSM = 0.4; DevURSM = 0.45;
```

```
In[641]:= PredUMean  = MeanUAnal; PredUDev = DevUAnal;
          {PredUMean, PredUDev} // N
```

```
Out[642]= {0., 0.05}

In[643]:= PredUMean = MeanUAnal + MeanURSM;
          PredUDev = Sqrt[DevUAnal^2 + DevURSM^2];
          {PredUMean, PredUDev} // N

Out[645]= {0.4, 0.452769}
```

This defines the change cost per part. The change of the height is expensive, as this is assumed to require a recutting of the extrusion orifice tool. The change of the modulus is less expensive, as only the material is changed. Note that the change cost is in $ per part. The failure cost is the cost occurring if the design model fails to produce a feasible design. This would require a different design approach or the expansion of the current design approach (i.e. the considerations of beam heights larger than the upper constraint limit or a modulus larger than the upper constraint limit, or a different beam cross-section)

```
In[646]:= ChangeHCost = 0.3; ChangeECost = 0.001; FailureCost =
```

Defines the resolution of the exhaustive search algorithm.

```
In[647]:= Resolution = 101;
```

Sets up the values of the initial design with the height *HA* and the modulus *EA*.

```
In[648]:= HA = 34.8;
          EA = 182150;
```

Determines the Upper Specification Limit under Uncertainty *USLU*. This defines the quality requirement, where the mean response has to be at least Alpha standard deviation from the upper specification limit.

```
In[650]:= H = HA; EMod = EA;
          USLU = USL - Alpha * DeflN;
          SetPrecision[USLU, 4]
          ClearAll[H, EMod];

Out[652]= 2.899
```

Sets up the Uncertainty Distributions and determines the Probability of satisfying the Quality Requirement, i.e. the upper specification limit under uncertainty *USLU*. as a functionof the height *H* and the modulus *EMod*.

```
In[654]:= ClearAll[H, EMod];
          DeflUDist =
           NormalDistribution[Defl + PredUMean, PredUDev];
          Yield = CDF[DeflUDist, USLU]
```

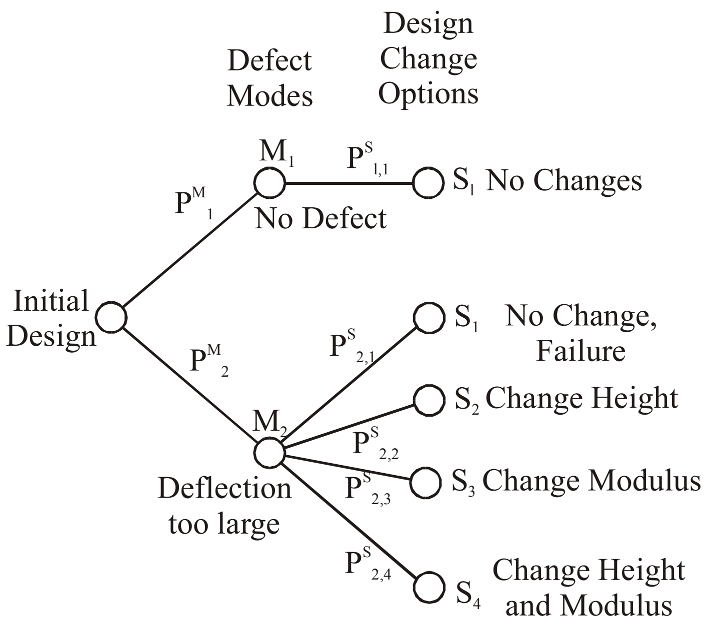
Out[656]= $\frac{1}{2}\left(1 + \text{Erf}[1.56174\,(-30.3019 + 0.000093542\,\text{EMod} - 9.16745 \times 10^{-11}\,\text{EMod}^2\;\; 0.865935\,H - 1.0741 \times 10^{-6}\,\text{EMod}\,H - 0.0061231\,H^2)]\right)$

Possible Expected Outcomes

There are theoretical three expected outcomes possible. The design cannot violate any constraint, it can violate the lower specification limit under uncertainty, and it can violate the upper specification limit under uncertainty. Since there exists no lower specification limit, this case can be eliminated, and only two cases remain. The probability of each case occurring is evaluated in the flexible design analysis.

Possible Design Changes

Since there are two design variables, there are four possible design changes. It is possible, not to change the design at all, it is possible to change only the height, it is possible to change only the modulus, and it is possible to change both the height and the modulus. The figure below shows the possible expected outcomes and the possible design changes for each expected outcome. Each of the branches of this bayesian network will be evaluated in more detail below. The results are shown as a table, where each row corresponds to one end of the branches, in the same order as in the graph. The following flexible design methodology determines the design changes with the largest probability of resolving a given expected outcome. The probability of resolving the defect, the probability of selecting a design change from the possible changes for a given expected outcome and the overall probability of a design change occurring is evaluated. This probability of a design change occurring is combined with the cost of this design including the design change.

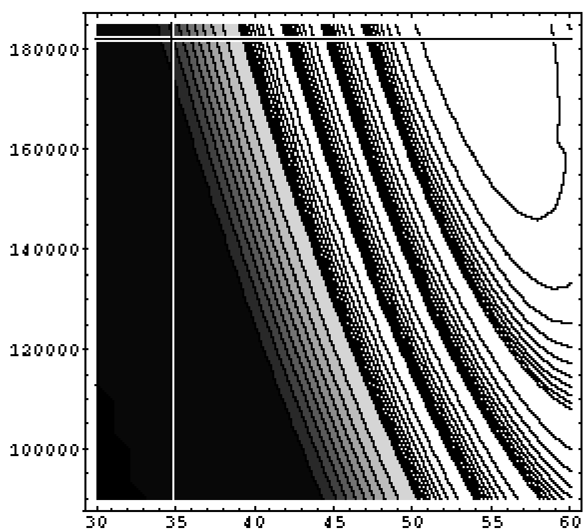


Design Optimizations

The design optimizations optimize the design for the different sub spaces. If there is no change, then there is no optimization necessary. This optimization determines the likelihood of satisfying the upper specification limit under uncertainty.The optimization normally depends on the defct mode, however, in this simple case with only one expected outcome it is posible to optimize the design merely for the probability of satisfying the specification under uncertainty.

Below is the optimization for the three other sub spaces, where only the height, only the modulus, or both the height and the modulus is changed. The section below creates a contour plot of the probability of satisfying the quality requirement, using the USLU for the given initial design. Dark areas have low probability of satisfying the quality requirement, light areas have a high probability of satisfying the quality requirements. The contour lines are not spaced evenly to enhance the details in the areas with high quality. The x-axis represents the height and the y-axis represents the modulus. This contour plot can be used for a manual search for an optimal design for each sub space. The initial design is shown with the black lines with white frames.

```
In[657]:= XW = Graphics[{AbsoluteThickness[3],
             RGBColor[1, 1, 1], Line[{{HL, EA}, {HH, EA}}]}];
          YW = Graphics[{AbsoluteThickness[3],
             RGBColor[1, 1, 1], Line[{{HA, EL}, {HA, EH}}]}];
          XB = Graphics[{AbsoluteThickness[1],
             RGBColor[0, 0, 0], Line[{{HL, EA}, {HH, EA}}]}];
          YB = Graphics[{AbsoluteThickness[1],
             RGBColor[0, 0, 0], Line[{{HA, EL}, {HA, EH}}]}];
          CPlot = ContourPlot[Yield, {H, HL, HH}, {EMod, EL, EH},
            Contours → {0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .91,
              .92, .93, .94, .95, .96, .97, .98, .99, .991,
              .992, .993, .994, .995, .996, .997, .998, .999,
              .9991, .9992, .9993, .9994, .9995, .9996, .9997,
              .9998, .9999, .99991, .99992, .99993, .99994,
              .99995, .99996, .99997, .99998, .99999, 1},
            PlotPoints → 30, DisplayFunction → Identity];
          Show[CPlot, XW, YW, XB, YB,
           DisplayFunction → $DisplayFunction];
```



Determines the Optimal Yield by changing the height *H*, also determines the optimal height. This is the one dimensional sub space where only the height can be changed. The search is exhaustive.

```
In[663]:= ClearAll[H, EMod];
          EMod = EA;
          YieldTable = Table[Yield,
             {H, HL, HH, ((HH - HL) / (Resolution - 1))}] // N;
          MaxYield = Max[YieldTable] // N;
          YieldPos = Position[YieldTable, Max[MaxYield]];
          HB = HL + ((HH - HL) / (Resolution - 1) *
               (YieldPos[[1, 1]] - 1)) // N;
          Res = Table[0, {i, 3}, {j, 2}];
          Res[[1, 1]] = "Optimal Yield";
          Res[[2, 1]] = "Height";
          Res[[3, 1]] = "Modulus";
          Res[[1, 2]] = MaxYield;
          Res[[2, 2]] = HB;
          Res[[3, 2]] = EA;
          MatrixForm[SetPrecision[Res, 4]]
          ClearAll[H, EMod];

Out[676]//MatrixForm=
```

$$\begin{pmatrix} \text{Optimal Yield} & 1.00 \\ \text{Height} & 54.6 \\ \text{Modulus} & 1.822\times10^{5} \end{pmatrix}$$

Determines the optimal yield by changing the modulus *EMod*, also determines the optimal modulus. This is the one dimensional sub space where only the modulus can be changed. The search is exhaustive.

```
In[678]:= ClearAll[H, EMod];
H = HA;
YieldTable = Table[Yield,
   {EMod, EL, EH, ((EH - EL) / (Resolution - 1))}] // N;
MaxYield = Max[YieldTable] // N;
YieldPos = Position[YieldTable, MaxYield];
EC = EL + ((EH - EL) / (Resolution - 1) *
     (YieldPos[[1, 1]] - 1)) // N;
Res = Table[0, {i, 3}, {j, 2}];
Res[[1, 1]] = "Optimal Yield";
Res[[2, 1]] = "Height";
Res[[3, 1]] = "Modulus";
Res[[1, 2]] = MaxYield;
Res[[2, 2]] = HA;
Res[[3, 2]] = EC;
MatrixForm[SetPrecision[Res, 4]]
ClearAll[H, EMod];
```

Out[691]//MatrixForm=

$$\begin{pmatrix} \text{Optimal Yield} & 0.2330 \\ \text{Height} & 34.8 \\ \text{Modulus} & 1.850 \times 10^5 \end{pmatrix}$$

Determines the optimal yield by changing the height and the modulus, also determines the optimal height and modulus. The table yield table has a increasing emod for the columns and an increasing height for the rows. This is the two-dimensional sub space where both the height and the modulus can be changed. The search is exhaustive.

```
In[693]:= ClearAll[H, EMod];
YieldTable =
 Table[Yield, {H, HL, HH, ((HH - HL) / (Resolution - 1))}
   {EMod, EL, EH, ((EH - EL) / (Resolution - 1))}] // N;
MaxYield = Max[YieldTable] // N;
YieldPos = Position[YieldTable, MaxYield];
HD = HL + ((HH - HL) / (Resolution - 1) *
      (YieldPos[[1, 1]] - 1)) // N;
ED = EL + ((EH - EL) / (Resolution - 1) *
      (YieldPos[[1, 2]] - 1)) // N;
Res = Table[0, {i, 3}, {j, 2}];
Res[[1, 1]] = "Optimal Yield";
Res[[2, 1]] = "Height";
Res[[3, 1]] = "Modulus";
Res[[1, 2]] = MaxYield;
Res[[2, 2]] = HD;
Res[[3, 2]] = ED;
MatrixForm[SetPrecision[Res, 4]]
ClearAll[H, EMod];
```

Out[706]//MatrixForm=

$$\begin{pmatrix} \text{Optimal Yield} & 1.00 \\ \text{Height} & 54.6 \\ \text{Modulus} & 1.850\times 10^{5} \end{pmatrix}$$

Flexible Design Analysis

Sets up the required variables for the different cases (1): No change, (2) Defect, no change, failure, (3) Defect, Change Height, (4) Defect, Change width, (5) Defect, Change both

These elements are in the same order as the branches of the graph above at the design change options. The matrices below are the height and the modulus for the five design options as optimized above.

```
In[708]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
MatrixForm[SetPrecision[H, 4]]
 MatrixForm[SetPrecision[EMod, 4]]
ClearAll[H, EMod];
```

```
Out[710]=
```

$$\begin{pmatrix} 34.8 \\ 34.8 \\ 54.6 \\ 34.8 \\ 54.6 \end{pmatrix} \begin{pmatrix} 1.822\times 10^5 \\ 1.822\times 10^5 \\ 1.822\times 10^5 \\ 1.850\times 10^5 \\ 1.850\times 10^5 \end{pmatrix}$$

Calculates the cost based on the change cost and the marginal part cost. Note, that the second cost is the failure cost as defined above. The cost is shown below.

```
In[712]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          MatrixForm[SetPrecision[Cost, 3]]
          ClearAll[H, EMod];

Out[714]//MatrixForm=
```

$$\begin{pmatrix} 1.64 \\ 1.64 \\ 2.03 \\ 1.65 \\ 2.03 \end{pmatrix}$$

This shows the change cost.

```
In[716]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          ChangeCost = {{0}, {0}, {ChangeHCost},
            {ChangeECost}, {ChangeECost + ChangeHCost}};
          MatrixForm[SetPrecision[ChangeCost, 3]]
          ClearAll[H, EMod];

Out[719]//MatrixForm=
```

$$\begin{pmatrix} 0 \\ 0 \\ 0.300 \\ 0.00100 \\ 0.301 \end{pmatrix}$$

This shows the total cost, Note, that the second case is the failure cost and defined separately.

```
In[721]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          TotalCost = ChangeCost + Cost // N;
          TotalCost[[2, 1]] = FailureCost;
          MatrixForm[SetPrecision[TotalCost, 3]]
          ClearAll[H, EMod];
```

Out[725]//MatrixForm=

$$\begin{pmatrix} 1.64 \\ 5.0 \\ 2.33 \\ 1.65 \\ 2.33 \end{pmatrix}$$

Probability of defect occurring. The *yield[[1,1]]* is the yield for the initial design, i.e. the first case, representing the likelihood of the defect occurring

```
In[727]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          PDefect =
           {{Yield[[1, 1]]}, {1 - Yield[[1, 1]]}, {1 - Yield[[1, 1
             {1 - Yield[[1, 1]]}, {1 - Yield[[1, 1]]}} // N;
          MatrixForm[SetPrecision[PDefect, 3]]
          ClearAll[H, EMod];
```

Out[730]//MatrixForm=

$$\begin{pmatrix} 0.192 \\ 0.81 \\ 0.81 \\ 0.81 \\ 0.81 \end{pmatrix}$$

Probability of satisfying the specifications after a design change. The first case is for no defect, therefore there will be no defects and there is certainty of no defects occurring. The second case is for defect, but no change, i.e. design failure, therefore a zero probability. This probability is not used elsewhere in the method. For the last three cases, the probability of satisfaction is calculated as the percentage of defects which occurred in the initial design, but do not occur in the current design, i.e. it is the relative improvement of the design. This is based on the conditional change in the probability distribution depending on the dfect mode. The If clause ensures that there is no division by zero (i.e. zero probability of defect occurring). If the probability of the defect occurring is zero, then this expected outcome will never happen , and all subsequent costs and probabilities are zero. Note, that the equations below are simplified from the main body of the idssertation due to the fact that only a one sided specificationis used.

```
In[732]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
  EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
  PResolve = Table[0, {a, 5}, {b, 1}];
  PResolve[[1, 1]] = 1;
  PResolve[[2, 1]] = 0;
  PResolve[[3, 1]] = If[PDefect[[3, 1]] == 0,
    1, 1 - ((1 - Yield[[3, 1]]) / PDefect[[3, 1]])];
  PResolve[[4, 1]] = If[PDefect[[4, 1]] == 0,
    1, 1 - ((1 - Yield[[4, 1]]) / PDefect[[4, 1]])];
  PResolve[[5, 1]] = If[PDefect[[5, 1]] == 0,
    1, 1 - ((1 - Yield[[5, 1]]) / PDefect[[5, 1]])];
  MatrixForm[SetPrecision[PResolve, 3]]
  ClearAll[H, EMod];

Out[740]//MatrixForm=
```

$$\begin{pmatrix} 1.00 \\ 0 \\ 1.0 \\ 0.051 \\ 1.0 \end{pmatrix}$$

Probability of Selecting a change from the list of changes for a given expected outcome. For the first case, selection can be ensured as the unchanged design will be used. For the last three cases:, the most economic case will be selected, the second most economic case only if it has excess likelihood of resolving, and the third most only if the two other have a smaller likelihood of resolving. The second case is the probability of failure for the given expected outcome, i.e. the remainder between the three last cases and certainty.

```
In[742]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          PSelect = {{1}, {0}, {0}, {0}, {0}};
          MinCostPos = Position[TotalCost[[{3, 4, 5}]],
              Min[TotalCost[[{3, 4, 5}]]]][[1, 1]] + 2;
          MaxCostPos = Position[TotalCost[[{3, 4, 5}]],
              Max[TotalCost[[{3, 4, 5}]]]][[1, 1]] + 2;
          MedCostPos = 12 - (MinCostPos + MaxCostPos);
          PSelect[[MinCostPos, 1]] = PResolve[[MinCostPos, 1]];
          PSelect[[MedCostPos, 1]] = Max[PResolve[[MedCostPos, :
             PResolve[[MinCostPos, 1]], 0];
          PSelect[[MaxCostPos, 1]] = Max[PResolve[[MaxCostPos, :
             Max[PResolve[[MinCostPos, 1]],
              PResolve[[MedCostPos, 1]]], 0];
          PSelect[[2, 1]] =
           1 - (PSelect[[3, 1]] + PSelect[[4, 1]] + PSelect[[5, 1]]
          MatrixForm[SetPrecision[PSelect, 3]]
          ClearAll[H, EMod];

Out[752]//MatrixForm=
```

$$\begin{pmatrix} 1.00 \\ 4.0\times10^{-6} \\ 0.95 \\ 0.051 \\ 1.49\times10^{-7} \end{pmatrix}$$

The Probability of a change occurring is the probability of the change being selected and the probability of the expected outcome occurring.

```
In[754]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          PChange = PSelect * PDefect;
          MatrixForm[SetPrecision[PChange, 3]]
          ClearAll[H, EMod];

Out[757]//MatrixForm=
```

$$\begin{pmatrix} 0.192 \\ 3.2\times10^{-6} \\ 0.77 \\ 0.041 \\ 1.21\times10^{-7} \end{pmatrix}$$

The individual parts of the expected cost are the total part cost times the probability of occurring.

```
In[759]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          CostE = PChange * TotalCost;
          MatrixForm[SetPrecision[CostE, 3]]
          ClearAll[H, EMod];
```

Out[762]//MatrixForm=

$$\begin{pmatrix} 0.316 \\ 0.0000161 \\ 1.79 \\ 0.068 \\ 2.82 \times 10^{-7} \end{pmatrix}$$

The Expected cost is then simply the sum of the individual costs and their occurrence.

```
In[764]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          ExpectedCost = Sum[CostE[[a, 1]], {a, 5}];
          SetPrecision[ExpectedCost, 4]
          ClearAll[H, EMod];

Out[767]= 2.170
```

Gives an overview report of the flexible design evaluation.

```
In[769]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          FlexDesReport = Table[0, {i, 1, 9}, {j, 1, 2}];
          VIC = {{"Expected Cost", "H", "EMod",
             "P(NO Change)", "P(Any Change)", "P(Failure)",
             "P(Change H)", "P(Change E)", "USLU"}};
          FlexDesReport[[All, 1]] = VIC[[1, All]];
          FlexDesReport[[1, 2]] = ExpectedCost;
          FlexDesReport[[2, 2]] = HA;
          FlexDesReport[[3, 2]] = EA;
          FlexDesReport[[4, 2]] = PChange[[1, 1]];
          FlexDesReport[[5, 2]] =
           PChange[[3, 1]] + PChange[[4, 1]] + PChange[[5, 1]];
          FlexDesReport[[6, 2]] = PChange[[2, 1]];
          FlexDesReport[[7, 2]] = PChange[[3, 1]] + PChange[[5, 1
          FlexDesReport[[8, 2]] = PChange[[4, 1]] + PChange[[5, 1
          FlexDesReport[[9, 2]] = USLU;
          FlexDesReport = SetPrecision[FlexDesReport, 4];
          MatrixForm[SetPrecision[FlexDesReport, 4]]
          ClearAll[H, EMod];
```

Out[784]//MatrixForm=

Expected Cost	2.170
H	34.8
EMod	1.822×10^5
P(NO Change)	0.1920
P(Any Change)	0.808
P(Failure)	3.23×10^{-6}
P(Change H)	0.767
P(Change E)	0.0410
USLU	2.899

This gives the table with all expected outcomes and design change combinations. Due to the number of columns, the table is split into multiple tables for printability. This table contains the different values calculated above.

```
In[786]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
VI = {"Notes", "H", "EMod", "Defl", "Cost",
  "ChangeCost", "Total Cost", "P(Defect)", "Yield",
  "P(Resolve)", "P(Select)", "P(Change)", "Exp.Cost"}
Comments = {{"OK, No Change"},
  {"Def, Failure"}, {"Def, Change H"},
  {"Def, Change E"}, {"Def, Change HE"}};
FlexDesEval =
 Transpose[{Transpose[Comments][[1]], Transpose[H][[
   Transpose[EMod][[1]], Transpose[Defl][[1]],
   Transpose[Cost][[1]], Transpose[ChangeCost][[1]],
   Transpose[TotalCost][[1]],
   Transpose[PDefect][[1]], Transpose[Yield][[1]],
   Transpose[PResolve][[1]], Transpose[PSelect][[1]]
   Transpose[PChange][[1]], Transpose[CostE][[1]]}];
FlexDesEvalA = {VI, FlexDesEval[[1]], FlexDesEval[[2]
  FlexDesEval[[3]], FlexDesEval[[4]], FlexDesEval[[5
FlexDesEvalA = SetPrecision[FlexDesEvalA, 4];
SetPrecision[
 MatrixForm[FlexDesEvalA[[All, {1, 2, 3, 4, 5}]]], 4]
SetPrecision[
 MatrixForm[FlexDesEvalA[[All, {1, 6, 7, 8, 9}]]], 4]
SetPrecision[
 MatrixForm[FlexDesEvalA[[All, {1, 10, 11, 12, 13}]]],
ClearAll[H, EMod];
```

Out[793]//MatrixForm=

Notes	H	EMod	Defl	Cost
OK, No Change	34.8	1.822×10^5	2.893	1.645
Def, Failure	34.8	1.822×10^5	2.893	1.645
Def, Change H	54.6	1.822×10^5	0.460	2.029
Def, Change E	34.8	1.850×10^5	2.829	1.649
Def, Change HE	54.6	1.850×10^5	0.457	2.033

Out[794]//MatrixForm=

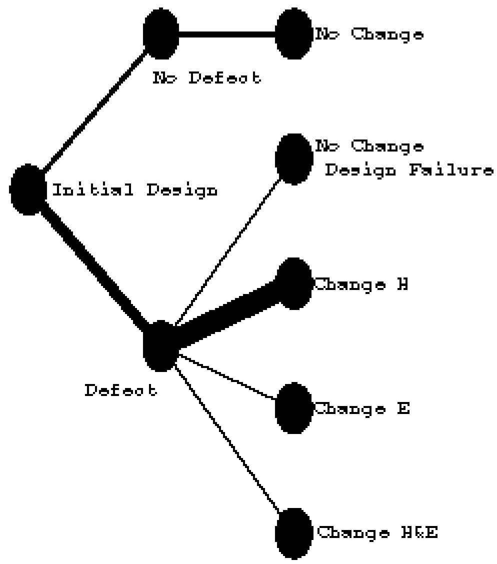
Notes	ChangeCost	Total Cost	P(Defect)	Y
OK, No Change	0	1.645	0.1920	0.
Def, Failure	0	5.00	0.808	0.
Def, Change H	0.3000	2.329	0.808	1
Def, Change E	0.001000	1.650	0.808	0.
Def, Change HE	0.3010	2.334	0.808	1

Out[795]//MatrixForm=

Notes	P(Resolve)	P(Select)	P(Change)	Ex
OK, No Change	1.00	1.00	0.192	[illegible]
Def, Failure	0	4.0×10^{-6}	3.2×10^{-6}	0.[illegible]
Def, Change H	1.0	0.95	0.77	
Def, Change E	0.051	0.051	0.041	[illegible]
Def, Change HE	1.0	1.49×10^{-7}	1.21×10^{-7}	2.[illegible]

This gives a graphical representation of the possible design changes, where the line thickness represents the likelihood of a certain defect or design change occurring.

```
In[797]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
OS = 15; th = 0.2; PA = {0, -1.25}; PB = {1.5, 0};
PC = {3, 0}; PD = {1.5, -2.5}; PE = {3, -1}; PF = {3, -2};
PG = {3, -3}; PH = {3, -4}; PI = {6, 0}; POffX = {-1.3, 0}
LA = {AbsoluteThickness[PDefect[[1, 1]] * OS],
  Line[{PA, PB, PC}]}; LB =
 {AbsoluteThickness[PDefect[[2, 1]] * OS], Line[{PA, PI
LC =
 {AbsoluteThickness[PChange[[2, 1]] * OS], Line[{PD, PE
LD =
 {AbsoluteThickness[PChange[[3, 1]] * OS], Line[{PD, PE
LE =
 {AbsoluteThickness[PChange[[4, 1]] * OS], Line[{PD, PG
LF =
 {AbsoluteThickness[PChange[[5, 1]] * OS], Line[{PD, PH
CA = {Disk[PA, th]}; CB = {Disk[PB, th]};
CC = {Disk[PC, th]}; CD = {Disk[PD, th]};
CE = {Disk[PE, th]}; CF = {Disk[PF, th]};
CG = {Disk[PG, th]}; CH = {Disk[PH, th]}; CI = {Disk[PI,
TA = Text["Initial Design", PA, {-1.2, 0}];
TB = Text["No Defect", PB, {-.8, 3.5}];
TC = Text["No Change", PC, POffX];
TD = Text["Defect", PD, {1, 3.5}];
TE = Text["No Change", PE, POffX + {0, -1}];
TEb = Text["Design Failure", PE, POffX + {0, 1}];
TF = Text["Change H", PF, POffX];
TG = Text["Change E", PG, POffX];
TH = Text["Change H&E", PH, POffX];
Show[Graphics[{LA, LB, LC, LD, LE,
   LF, CA, CB, CC, CD, CE, CF, CG, CH, CI, TA, TB,
   TC, TD, TE, TEb, TF, TG, TH}, AspectRatio -> 1]];
ClearAll[H, EMod];
```



Previously Calculated Flexible Design Evaluations

The values shown below are simply CutCopy from the results summary of the flexible design evaluation above.

Large Uncertainty

Shows different design points for designs with the large uncertainty from the RSM model.

In[4311]:= Results for Minimum Marginal Cost, using large uncertainty

Expected Cost	2.170
H	34.8
EMod	1.822×10^5
P(NO Change)	0.1920
P(Any Change)	0.808
P(Failure)	3.23×10^{-6}
P(Change H)	0.767
P(Change E)	0.0410
USLU	2.899

Notes	H	EMod	Defl	Cost
OK, No Change	34.8	1.822×10^5	2.893	1.645
Def, Failure	34.8	1.822×10^5	2.893	1.645
Def, Change H	54.6	1.822×10^5	0.460	2.029
Def, Change E	34.8	1.850×10^5	2.829	1.649
Def, Change HE	54.6	1.850×10^5	0.457	2.033

Notes	ChangeCost	Total Cost	P(Defect)	Yield
OK, No Change	0	1.645	0.1920	0.1920
Def, Failure	0	5.00	0.808	0.1920
Def, Change H	0.3000	2.329	0.808	1.00
Def, Change E	0.001000	1.650	0.808	0.2330
Def, Change HE	0.3010	2.334	0.808	1.00

Notes	P(Resolve)	P(Select)	P(Change)	Exp.Cost
OK, No Change	1.000	1.000	0.1920	0.3158
Def, Failure	0	3.99×10^{-6}	3.23×10^{-6}	0.00001613
Def, Change H	1.00	0.949	0.767	1.786
Def, Change E	0.0508	0.0508	0.0410	0.0677
Def, Change HE	1.00	1.493×10^{-7}	1.206×10^{-7}	$2.816 \times 10^{-}$

In[4311]:= Optimal Results for Minimum Expected Cost, using large uncertainty

Expected Cost	1.744
H	40.9
EMod	1.490×10^5
P(NO Change)	0.702
P(Any Change)	0.2976
P(Failure)	2.855×10^{-6}
P(Change H)	0.02060
P(Change E)	0.2770
USLU	2.911

Notes	H	EMod	Defl	Cost
OK, No Change	40.9	1.490×10^5	2.270	1.717
Def, Failure	40.9	1.490×10^5	2.270	1.717
Def, Change H	57.6	1.490×10^5	0.554	2.033
Def, Change E	40.9	1.850×10^5	1.587	1.767
Def, Change HE	54.6	1.850×10^5	0.457	2.033

Notes	ChangeCost	Total Cost	P(Defect)	Yield
OK, No Change	0	1.717	0.702	0.702
Def, Failure	0	5.00	0.2976	0.702
Def, Change H	0.3000	2.333	0.2976	1.00
Def, Change E	0.001000	1.768	0.2976	0.979
Def, Change HE	0.3010	2.334	0.2976	1.00

Notes	P(Resolve)	P(Select)	P(Change)	Exp.Cost
OK, No Change	1.000	1.000	0.702	1.206
Def, Failure	0	9.59×10^{-6}	2.855×10^{-6}	0.00001428
Def, Change H	1.00	0.0692	0.02060	0.0481
Def, Change E	0.931	0.931	0.2770	0.490
Def, Change HE	1.00	0.00001640	4.88×10^{-6}	0.00001139

Small Uncertainty for RSM model

Minimum Expected Cost under small uncertainty

Expected Cost	1.651
H	35.1
EMod	1.810×10^5
P(NO Change)	0.861
P(Any Change)	0.1387
P(Failure)	0
P(Change H)	0.002058
P(Change E)	0.1387
USLU	2.901

Notes	H	EMod	Defl	Cost
OK, No Change	35.1	1.810×10^5	2.847	1.649
Def, Failure	35.1	1.810×10^5	2.847	1.649
Def, Change H	36.9	1.810×10^5	2.431	1.684
Def, Change E	35.1	1.850×10^5	2.757	1.654
Def, Change HE	36.3	1.850×10^5	2.481	1.678

Notes	ChangeCost	Total Cost	P(Defect)	Yield
OK, No Change	0	1.649	0.861	0.861
Def, Failure	0	5.00	0.1387	0.861
Def, Change H	0.3000	1.984	0.1387	1.000
Def, Change E	0.001000	1.655	0.1387	0.998
Def, Change HE	0.3010	1.979	0.1387	1.000

Notes	P(Resolve)	P(Select)	P(Change)	Exp.Cost
OK, No Change	1.000	1.000	0.861	1.420
Def, Failure	0	0	0	0
Def, Change H	1.000	0	0	0
Def, Change E	0.985	0.985	0.1367	0.2262
Def, Change HE	1.000	0.01484	0.002058	0.00407

ANALYTICAL Model, Small Uncertainty

In[4311]:= Optimal Results for Minimum Expected Cost using ANALYTICAL MODE

Expected Cost	1.614
H	33.8
EMod	1.720×10^5
P(NO Change)	0.900
P(Any Change)	0.0997
P(Failure)	0
P(Change H)	7.11×10^{-8}
P(Change E)	0.0997
USLU	2.895

Notes	H	EMod	Defl	Cost
OK, No Change	33.8	1.720×10^5	2.831	1.613
Def, Failure	33.8	1.720×10^5	2.831	1.613
Def, Change H	35.7	1.720×10^5	2.472	1.649
Def, Change E	33.8	1.850×10^5	2.632	1.629
Def, Change HE	34.8	1.831×10^5	2.474	1.646

Notes	ChangeCost	Total Cost	P(Defect)	Yield
OK, No Change	0	1.613	0.900	0.900
Def, Failure	0	5.00	0.0997	0.900
Def, Change H	0.3000	1.949	0.0997	1.000
Def, Change E	0.001000	1.630	0.0997	1.00
Def, Change HE	0.3010	1.947	0.0997	1.000

Notes	P(Resolve)	P(Select)	P(Change)	Exp.Cost
OK, No Change	1.000	1.000	0.900	1.452
Def, Failure	0	0	0	0
Def, Change H	1.000	0	0	0
Def, Change E	1.00	1.00	0.0997	0.1625
Def, Change HE	1.000	7.13×10^{-7}	7.11×10^{-8}	1.384×10^{-7}

Flexible Design Methodology Including Design Change Uncertainty

Analyzes the flexible design methodology for a given initial design, assuming uncertainty in the design change.

Initial Design & Setup

It is possible e to use either the analytical models (*DeflAnal, DeflNAnal, CostAnal*) or the RSM models (*DeflMod, DeflNMod, CostMod*). The selection of the models is decided below by swapping between the two different definition cells shown below.

```
In[811]:= Defl = DeflAnal;
          DeflN = DeflNAnal;
          Cost = CostAnal;
```

```
In[814]:= Defl = DeflMod;
          DeflN = DeflNMod;
          Cost = CostMod;
```

This defines the mean and standard deviation of the uncertainty distribution for the deflection. Two sources of uncertainty can be used. The *MeanUAnal* and *DevUAnal* one is to represent the uncertainty of the analytical model, where for example holes for attachments or rounded corners can weaken the beam, whereas a welded attachment may strengthen the beam. As these effects are not included in the analytical model, there might be some prediction errors of the analytical model. The *MeanURSM* and the *DevURSM* represents the uncertainty of the response surface model due to fitting errors. Note, that these values differ from the absolute mean and standard deviation of the RSM model as calculated above, because the values below are derived from 10 random sample points.

If the analytical model is used, only the analytical uncertainty is used. If the RSM model is used, both the analytical uncertainty and the model fitting uncertainty are used. To switch between the uncertainty models, please move the appropriate sections to the end of this statement.

```
In[817]:= MeanUAnal = 0; DevUAnal  = 0.05;
          MeanURSM = 0.4; DevURSM = 0.45;
```

```
In[819]:= PredUMean  = MeanUAnal; PredUDev = DevUAnal;
          {PredUMean, PredUDev} // N

Out[820]= {0., 0.05}
```

```
In[821]:= PredUMean = MeanUAnal + MeanURSM;
          PredUDev = Sqrt[DevUAnal ^ 2 + DevURSM ^ 2];
          {PredUMean, PredUDev} // N
```

```
Out[823]= {0.4, 0.452769}
```

The uncertainty distributions of the design change is set to have a standard deviation of 0.1 mm.

```
In[824]:= DevUChange = 0.1;
```

This defines the change cost per part. The change of the height is expensive, as this is assumed to require a recutting of the extrusion orifice tool. The change of the modulus is less expensive, as only the material is changed. Note that the change cost is in $ per part. The failure cost is the cost occurring if the design model fails to produce a feasible design. This would require a different design approach or the expansion of the current design approach (i.e. the considerations of beam heights larger than the upper constraint limit or a modulus larger than the upper constraint limit, or a different beam cross-section)

```
In[825]:= ChangeHCost  = 0.3; ChangeECost  = 0.001; FailureCost  =
```

Defines the resolution of the exhaustive search algorithm.

```
In[826]:= Resolution = 101;
```

Sets up the values of the initial design with the height *HA* and the modulus *EA*.

```
In[827]:= HA =  34.8;
          EA  =  182200;
```

Determines the Upper Specification Limit under Uncertainty *USLU*. This defines the quality requirement, where the mean response has to be at least Alpha standard deviation from the upper specification limit.

```
In[829]:= H = HA;  EMod = EA;
          USLU = USL - Alpha * DeflN;
          SetPrecision[USLU, 4]
          ClearAll[H, EMod];

Out[831]= 2.899
```

Sets up the Uncertainty Distributions and determines the Probability of satisfying the Quality Requirement, i.e. the upper specification limit under uncertainty *USLU*. as a function of the height *H* and the modulus *EMod*.

```
In[833]:= ClearAll[H, EMod];
          DeflUDist =
           NormalDistribution[Defl + PredUMean, PredUDev];
          Yield = CDF[DeflUDist, USLU]
```

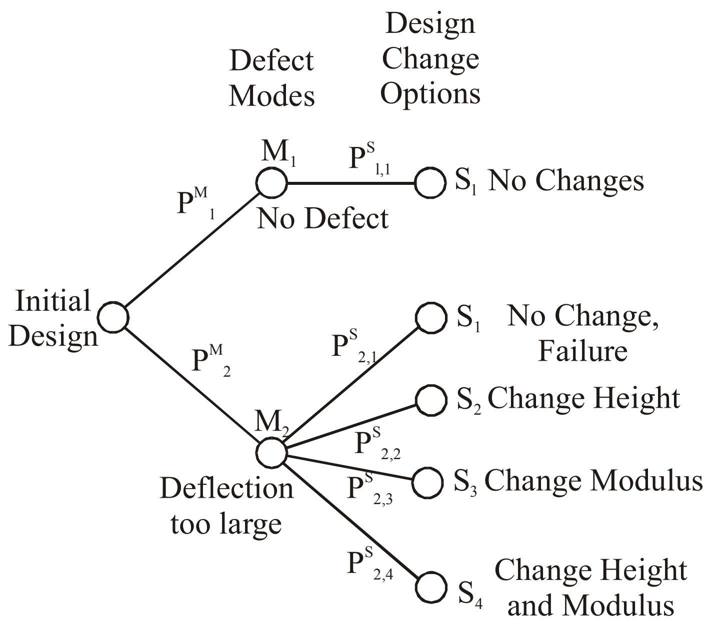
Out[835]= $\frac{1}{2}$ (1 + Erf[1.56174 (-30.3019 + 0.000093542 EMod - 9.16745 $\times 10^{-11}$ EMod^2 0.865935 H - 1.0741 $\times 10^{-6}$ EMod H - 0.0061231 H^2)])

Possible expected outcomes

There are theoretical three expected outcomes possible. The design cannot violate any constraint, it can violate the lower specification limit under uncertainty, and it can violate the upper specification limit under uncertainty. Since there exists no lower specification limit, this case can be eliminated, and only two cases remain. The probability of each case occurring is evaluated in the flexible design analysis.

Possible Design Changes

Since there are two design variables, there are four possible design changes. It is possible, not to change the design at all, it is possible to change only the height, it is possible to change only the modulus, and it is possible to change both the height and the modulus. The figure below shows the possible expected outcomes and the possible design changes for each expected outcome. Each of the branches of this bayesian network will be evaluated in more detail below. The results are shown as a table, where each row corresponds to one end of the branches, in the same order as in the graph. The following flexible design methodology determines the design changes with the largest probability of resolving a given expected outcome. The probability of resolving the defect, the probability of selecting a design change from the possible changes for a given expected outcome and the overall probability of a design change occurring is evaluated. This probability of a design change occurring is combined with the cost of this design including the design change.

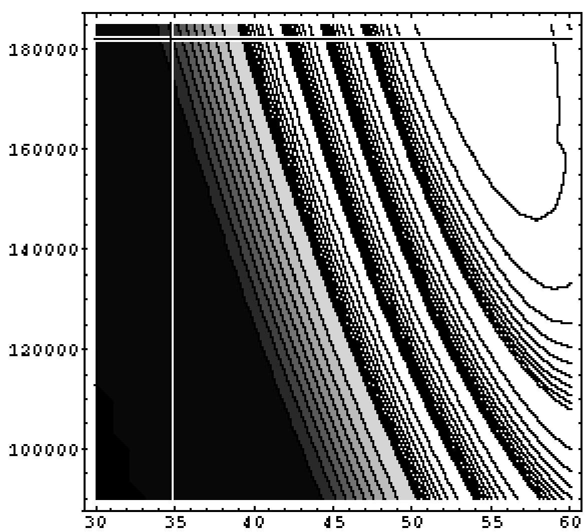


Design Optimizations

The design optimizations optimize the design for the different sub spaces. If there is no change, then there is no optimization necessary. This optimization determines the likelihood of satisfying the upper specification limit under uncertainty.The optimization normally depends on the defct mode, however, in this simple case with only one expected outcome it is posible to optimize the design merely for the probability of satisfying the specification under uncertainty.

Below is the optimization for the three other sub spaces, where only the height, only the modulus, or both the height and the modulus is changed. The section below creates a contour plot of the probability of satisfying the quality requirement, using the USLU for the given initial design. Dark areas have low probability of satisfying the quality requirement, light areas have a high probability of satisfying the quality requirements. The contour lines are not spaced evenly to enhance the details in the areas with high quality. The x-axis represents the height and the y-axis represents the modulus. This contour plot can be used for a manual search for an optimal design for each sub space. The initial design is shown with the black lines with white frames.

```
In[836]:= XW = Graphics[{AbsoluteThickness[3],
             RGBColor[1, 1, 1], Line[{{HL, EA}, {HH, EA}}]}];
          YW = Graphics[{AbsoluteThickness[3],
             RGBColor[1, 1, 1], Line[{{HA, EL}, {HA, EH}}]}];
          XB = Graphics[{AbsoluteThickness[1],
             RGBColor[0, 0, 0], Line[{{HL, EA}, {HH, EA}}]}];
          YB = Graphics[{AbsoluteThickness[1],
             RGBColor[0, 0, 0], Line[{{HA, EL}, {HA, EH}}]}];
          CPlot = ContourPlot[Yield, {H, HL, HH}, {EMod, EL, EH},
            Contours → {0, .1, .2, .3, .4, .5, .6, .7, .8, .9, .91,
              .92, .93, .94, .95, .96, .97, .98, .99, .991,
              .992, .993, .994, .995, .996, .997, .998, .999,
              .9991, .9992, .9993, .9994, .9995, .9996, .9997,
              .9998, .9999, .99991, .99992, .99993, .99994,
              .99995, .99996, .99997, .99998, .99999, 1},
            PlotPoints → 30, DisplayFunction → Identity];
          Show[CPlot, XW, YW, XB, YB,
           DisplayFunction → $DisplayFunction];
```



Determines the Optimal Yield by changing the height *H*, also determines the optimal height. This is the one dimensional sub space where only the height can be changed. The search is exhaustive.

```
In[842]:= ClearAll[H, EMod];
EMod = EA;
YieldTable = Table[Yield,
   {H, HL, HH, ((HH - HL) / (Resolution - 1))}] // N;
MaxYield = Max[YieldTable] // N;
YieldPos = Position[YieldTable, Max[MaxYield]];
HB = HL + ((HH - HL) / (Resolution - 1) *
     (YieldPos[[1, 1]] - 1)) // N;
Res = Table[0, {i, 3}, {j, 2}];
Res[[1, 1]] = "Optimal Yield";
Res[[2, 1]] = "Height";
Res[[3, 1]] = "Modulus";
Res[[1, 2]] = MaxYield;
Res[[2, 2]] = HB;
Res[[3, 2]] = EA;
MatrixForm[SetPrecision[Res, 4]]
ClearAll[H, EMod];

Out[855]//MatrixForm=
```

$$\begin{pmatrix} \text{Optimal Yield} & 1.00 \\ \text{Height} & 54.6 \\ \text{Modulus} & 1.822\times 10^{5} \end{pmatrix}$$

Determines the optimal yield by changing the modulus *EMod*, also determines the optimal modulus. This is the one dimensional sub space where only the modulus can be changed. The search is exhaustive.

```
In[857]:= ClearAll[H, EMod];
H = HA;
YieldTable = Table[Yield,
   {EMod, EL, EH, ((EH - EL) / (Resolution - 1))}] // N;
MaxYield = Max[YieldTable] // N;
YieldPos = Position[YieldTable, MaxYield];
EC = EL + ((EH - EL) / (Resolution - 1) *
     (YieldPos[[1, 1]] - 1)) // N;
Res = Table[0, {i, 3}, {j, 2}];
Res[[1, 1]] = "Optimal Yield";
Res[[2, 1]] = "Height";
Res[[3, 1]] = "Modulus";
Res[[1, 2]] = MaxYield;
Res[[2, 2]] = HA;
Res[[3, 2]] = EC;
MatrixForm[SetPrecision[Res, 4]]
ClearAll[H, EMod];
```

Out[870]//MatrixForm=

$$\begin{pmatrix} \text{Optimal Yield} & 0.2331 \\ \text{Height} & 34.8 \\ \text{Modulus} & 1.850\times 10^{5} \end{pmatrix}$$

Determines the optimal yield by changing the height and the modulus, also determines the optimal height and modulus. The table yield table has a increasing emod for the columns and an increasing height for the rows. This is the two-dimensional sub space where both the height and the modulus can be changed. The search is exhaustive.

In[872]:=
```
ClearAll[H, EMod];
YieldTable =
 Table[Yield, {H, HL, HH, ((HH - HL) / (Resolution - 1))},
  {EMod, EL, EH, ((EH - EL) / (Resolution - 1))}] // N;
MaxYield = Max[YieldTable] // N;
YieldPos = Position[YieldTable, MaxYield];
HD = HL + ((HH - HL) / (Resolution - 1) *
     (YieldPos[[1, 1]] - 1)) // N;
ED = EL + ((EH - EL) / (Resolution - 1) *
     (YieldPos[[1, 2]] - 1)) // N;
Res = Table[0, {i, 3}, {j, 2}];
Res[[1, 1]] = "Optimal Yield";
Res[[2, 1]] = "Height";
Res[[3, 1]] = "Modulus";
Res[[1, 2]] = MaxYield;
Res[[2, 2]] = HD;
Res[[3, 2]] = ED;
MatrixForm[SetPrecision[Res, 4]]
ClearAll[H, EMod];
```

Out[885]//MatrixForm=

$$\begin{pmatrix} \text{Optimal Yield} & 1.00 \\ \text{Height} & 54.6 \\ \text{Modulus} & 1.850 \times 10^5 \end{pmatrix}$$

Flexible Design Analysis

Sets up the required variables for the different cases (1): No change, (2) Defect, no change, failure, (3) Defect, Change Height, (4) Defect, Change width, (5) Defect, Change both

These elements are in the same order as the branches of the graph above at the design change options. The matrices below are the height and the modulus for the five design options as optimized above.

In[887]:=
```
H = {{HA}, {HA}, {HB}, {HA}, {HD}};
EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
MatrixForm[SetPrecision[H, 4]]
 MatrixForm[SetPrecision[EMod, 4]]
ClearAll[H, EMod];
```

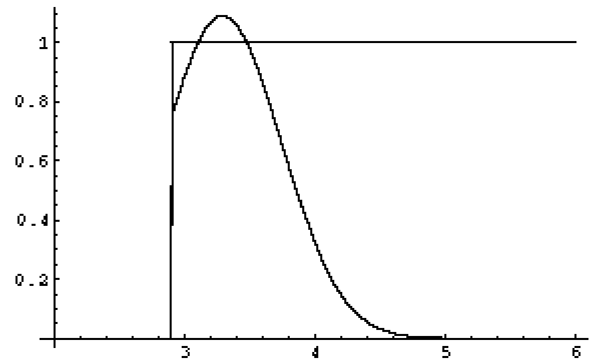
```
Out[889]= (34.8)  (1.822 × 10^5)
          (34.8)  (1.822 × 10^5)
          (54.6)  (1.822 × 10^5)
          (34.8)  (1.850 × 10^5)
          (54.6)  (1.850 × 10^5)
```

Probability of defect occurring. The *yield[[1,1]]* is the yield for the initial design, i.e. the first case, representing the likelihood of the defect occurring

```
In[891]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          PDefect =
           {{Yield[[1, 1]]}, {1 - Yield[[1, 1]]}, {1 - Yield[[1, 1
             {1 - Yield[[1, 1]]}, {1 - Yield[[1, 1]]}} // N;
          MatrixForm[SetPrecision[PDefect, 3]]
          ClearAll[H, EMod];

Out[894]//MatrixForm=
          (0.193)
          (0.81)
          (0.81)
          (0.81)
          (0.81)
```

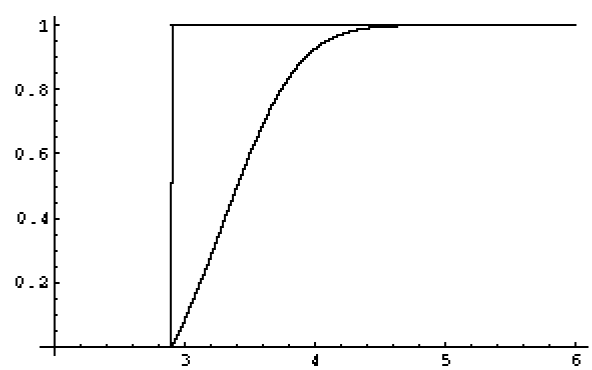
Probability density function of Conditional Defect

```
In[896]:= ClearAll[H, EMod, z, y];
          H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          CondDef =
           NormalDistribution[Defl[[1, 1]] + PredUMean, PredUDev
          z = y - USLU;
          CondDefDist =
           PDF[CondDef, z + USLU] / (1 - CDF[CondDef, USLU]);
          Plot[{CondDefDist * UnitStep[z], UnitStep[z]},
           {y, 2, 6}, PlotRange → All];
          Clear[z, y]
```



Cumulative density function Distribution of Conditional Defect

```
In[904]:= Clear[z, y]
          CDFCondDefDist =
           (CDF[CondDef, z + USLU] - CDF[CondDef, USLU]) /
            (1 - CDF[CondDef, USLU]);
          z = y - USLU;
          Plot[{CDFCondDefDist * UnitStep[z], UnitStep[z]},
           {y, 2, 6}, PlotRange → All];
          Clear[z, y]
```



Determines the possible specification violations which can be adjusted by means of the designchange deterministically.

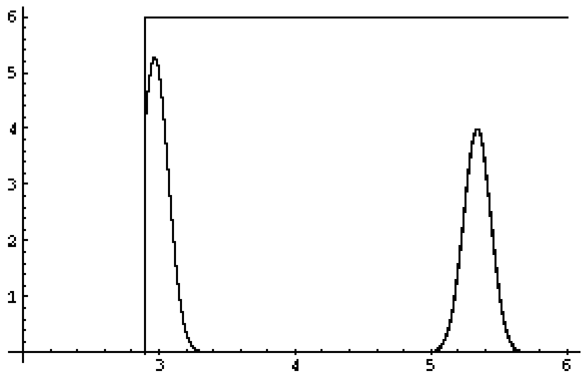
```
In[909]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
         EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
         DetChange = USLU - Defl;
         DetChange[[1, 1]] = 0;
         DetChange[[2, 1]] = 0;
         MatrixForm[SetPrecision[DetChange + USLU, 4]]
         ClearAll[H, EMod];
```

```
Out[914]//MatrixForm=
```

$$\begin{pmatrix} 2.899 \\ 2.899 \\ 5.34 \\ 2.969 \\ 5.34 \end{pmatrix}$$

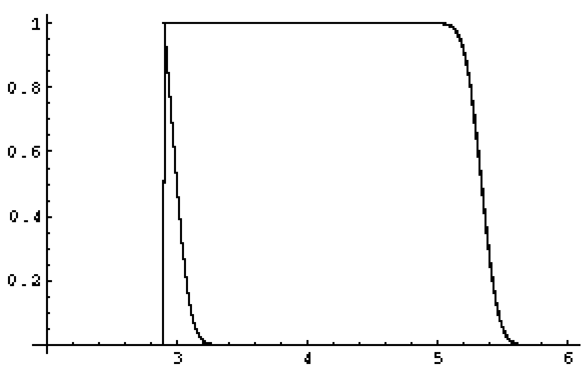
Determines the probability Distribution for the sucess of a design change based on the deterministic change limit and the given standard deviation. The deterministic limit represents the maximum likelihood point, i.e. the top of the uncertainty distribution Note, that the values less than the limit are cut off, and the distribution is scaled so that the area underneat the curve is zero.

```
In[916]:= Clear[z, y]
         H = {{HA}, {HA}, {HB}, {HA}, {HD}};
         EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
         VDevUChange = {DevUChange,
           DevUChange, DevUChange, DevUChange, DevUChange};
         ChangeDist =
          PDF[NormalDistribution[DetChange, VDevUChange], z] /
             CDF[NormalDistribution[DetChange, VDevUChange], (
         z = y - USLU;
         Plot[{ChangeDist[[3]], ChangeDist[[4]] * UnitStep[z],
           ChangeDist[[5]], UnitStep[z] * 6}, {y, 2, 6},
          PlotRange → All, DisplayFunction → $DisplayFunction];
         Clear[z, y]
```



Determines the cummulative Distribution for the sucess of a design change based on the above PDF.

```
In[924]:= Clear[y, z]
          PDFChangeDist = 1 - Integrate[ChangeDist, {z, 0, z}];
          z = y - USLU;
          Plot[{PDFChangeDist[[3]] * UnitStep[z],
            PDFChangeDist[[4]] * UnitStep[z],
            PDFChangeDist[[5]] * UnitStep[z]}, {y, 2, 6},
           PlotRange → All, DisplayFunction → $DisplayFunction];
          Clear[y, z]
```



Sorts the three possible design changes according to the max deterministic value of satisfaction.

```
In[929]:= PossibleDetChanges = {DetChange[[3, 1]],
            DetChange[[4, 1]], DetChange[[5, 1]]};
          SC = {1, 1, 1};
          SC[[1]] = (Position[PossibleDetChanges,
               Min[PossibleDetChanges]] + 2)[[1, 1]];
          SC[[3]] = (Position[PossibleDetChanges,
               Max[PossibleDetChanges]] + 2)[[1, 1]];
          SC[[2]] = 12 - SC[[1]] - SC[[3]];
          SC

Out[934]= {4, 3, 5}
```

Calculates the cost based on the change cost and the marginal part cost. Note, that the second cost is the failure cost as defined above. The cost is shown below.

```
In[935]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          MatrixForm[SetPrecision[Cost, 3]]
          ClearAll[H, EMod];

Out[937]//MatrixForm=
          (1.64)
          (1.64)
          (2.03)
          (1.65)
          (2.03)
```

This shows the change cost.

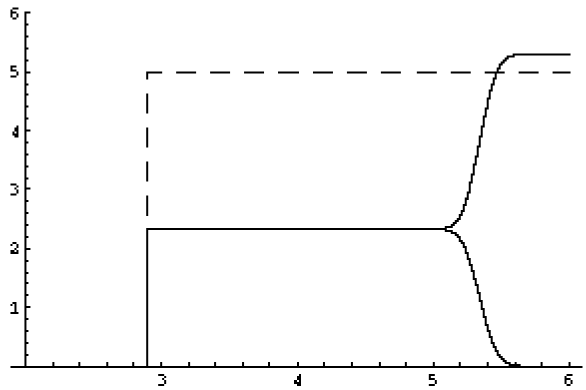
```
In[939]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
          EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
          ChangeCost = {{0}, {0}, {ChangeHCost},
            {ChangeECost}, {ChangeECost + ChangeHCost}};
          MatrixForm[SetPrecision[ChangeCost, 3]]
          ClearAll[H, EMod];
```

Out[942]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0.300 \\ 0.00100 \\ 0.301 \end{pmatrix}$$

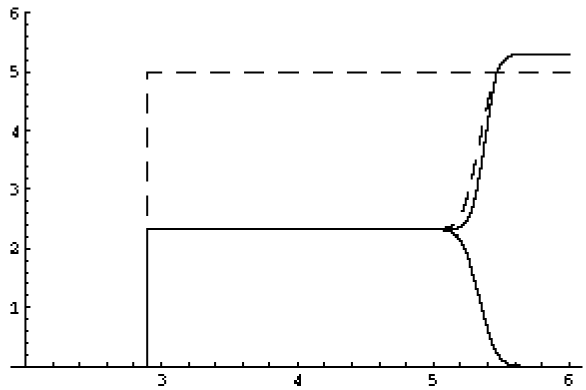
Determine Cost of most reliable change as a function of the encountered defect. The failure cost is also shown as a dashed line.

In[944]:=
```
ClearAll[y, z]
TotCost = {1, 1, 1};
SucessCost = TotCost;
FailCost = TotCost;
H = {{HA}, {HA}, {HB}, {HA}, {HD}};
EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
z = y - USLU;
CRA = SC[[3]];
SucessCost[[CRA - 2]] =
 ((Cost[[CRA]] + ChangeCost[[CRA]])[[1]] *
    PDFChangeDist[[CRA]])[[1]];
FailCost[[CRA - 2]] =
 ((ChangeCost[[CRA]] + FailureCost)[[1]] *
    (1 - PDFChangeDist[[CRA]]))[[1]];
TotCost[[CRA - 2]] =
 Simplify[SucessCost[[CRA - 2]] + FailCost[[CRA - 2]]];
Plot[
 {SucessCost[[CRA - 2]] * UnitStep[z], TotCost[[CRA - 2]]
   UnitStep[z], FailureCost * UnitStep[z]},
 {y, 2, 6}, PlotRange → {0, 6},
 DisplayFunction → $DisplayFunction, PlotStyle →
  {GrayLevel[0], GrayLevel[0], Dashing[{0.03}]}];
ClearAll[y, z]
```



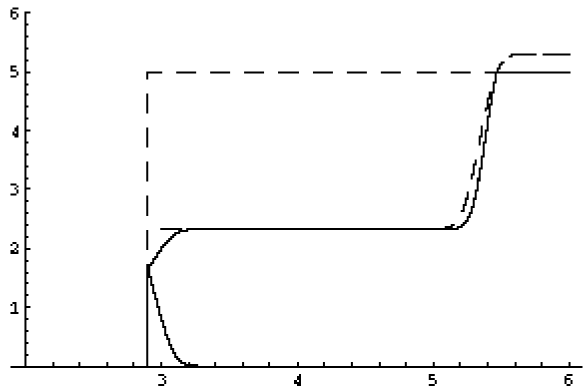
Determine Cost of second reliable change as a function of the encountered defect.The failure cost and the most reliable change are also shown as dashed lines.

```
In[957]:= ClearAll[y, z];
         H = {{HA}, {HA}, {HB}, {HA}, {HD}};
         EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
         CRB = SC[[2]];
         SucessCost[[CRB - 2]] =
          ((Cost[[CRB]] + ChangeCost[[CRB]])[[1]] *
             PDFChangeDist[[CRB]])[[1]];
         FailCost[[CRB - 2]] =
          ((ChangeCost[[CRB]] +
                Min[FailureCost, TotCost[[CRA - 2]]])[[1]] *
             (1 - PDFChangeDist[[CRB]]))[[1]];
         TotCost[[CRB - 2]] =
          Simplify[SucessCost[[CRB - 2]] + FailCost[[CRB - 2]]];
         z = y - USLU;
         Plot[{SucessCost[[CRB - 2]] * UnitStep[z],
           TotCost[[CRB - 2]] * UnitStep[z],
           TotCost[[CRA - 2]] * UnitStep[z],
           FailureCost * UnitStep[z]}, {y, 2, 6},
          PlotRange → {0, 6}, DisplayFunction → $DisplayFunctio
          PlotStyle → {GrayLevel[0], GrayLevel[0],
            Dashing[{0.03}], Dashing[{0.03}]}];
```



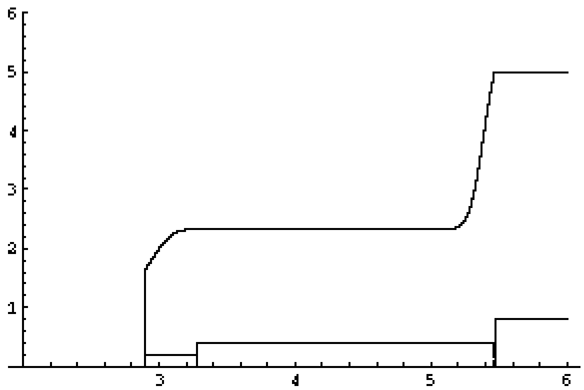
Determine Cost of thirdmost reliable change as a function of the encountered defect.The failure cost and the most and secondmost reliable change are also shown as dashed lines.

```
In[966]:= ClearAll[y, z];
    H = {{HA}, {HA}, {HB}, {HA}, {HD}};
    EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
    CRC = SC[[1]];
    SucessCost[[CRC - 2]] =
     ((Cost[[CRC]] + ChangeCost[[CRC]])[[1]] *
        PDFChangeDist[[CRC]])[[1]];
    FailCost[[CRC - 2]] =
     ((ChangeCost[[CRC]] + Min[FailureCost,
            TotCost[[CRA - 2]], TotCost[[CRB - 2]]])[[1]] *
        (1 - PDFChangeDist[[CRC]]))[[1]];
    TotCost[[CRC - 2]] =
     Simplify[SucessCost[[CRC - 2]] + FailCost[[CRC - 2]]];
    z = y - USLU;
    Plot[{SucessCost[[CRC - 2]] * UnitStep[z],
      TotCost[[CRC - 2]] * UnitStep[z],
      TotCost[[CRB - 2]] * UnitStep[z],
      TotCost[[CRA - 2]] * UnitStep[z],
      FailureCost * UnitStep[z]}, {y, 2, 6},
     PlotRange → {0, 6}, DisplayFunction → $DisplayFunctio
     PlotStyle → {GrayLevel[0], GrayLevel[0],
       Dashing[{0.03}], Dashing[{0.03}], Dashing[{0.03}]]
```



Plots the overall min cost for each possible defect. Also shows which change is the most efficient for which defect. Where the step function is at 0.2, the change in modulus is the best attempt. Where the step function is 0.4, the change in height is the best attempt. For 0.6 the change in height and modulus is the best attempt, and for 0.8, no change but a direct failure cost is the best solution.

```
In[1032]:= ClearAll[y, z];
         H = {{HA}, {HA}, {HB}, {HA}, {HD}};
         EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
         MinFun = {1, 1, 1, 1};
         MinFun[[1]] = (Sign[BestCost - TotCost[[1]]] + 1) * .4;
         MinFun[[2]] = (Sign[BestCost - TotCost[[2]]] + 1) * .2;
         MinFun[[3]] = (Sign[BestCost - TotCost[[3]]] + 1) * .6;
         MinFun[[4]] = (Sign[BestCost - FailureCost] + 1) * .8;
         BestCost = Min[FailureCost, TotCost[[CRA - 2]],
           TotCost[[CRB - 2]], TotCost[[CRC - 2]]];
         z = y - USLU;
         Plot[{BestCost * UnitStep[z], MinFun[[1]] * UnitStep[z
           MinFun[[2]] * UnitStep[z], MinFun[[3]] * UnitStep[z]
           MinFun[[4]] * UnitStep[z]}, {y, 2, 6},
          PlotRange -> {0, 6}, DisplayFunction -> $DisplayFunctio
```

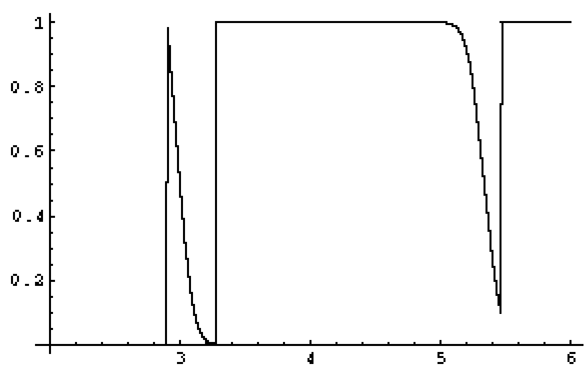


Finds the switchover points.

```
In[986]:= ClearAll[y, z];
         H = {{HA}, {HA}, {HB}, {HA}, {HD}};
         EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
         z = y - USLU;
         FindMinimum[
           (TotCost[[2]] - TotCost[[1]]) ^2, {y, 3, 3.1}][[2]]
         FindMinimum[
           (TotCost[[1]] - TotCost[[3]]) ^2, {y, 5.5, 5.6}][[2]]
         FindMinimum[(FailureCost - TotCost[[1]]) ^2, {y, 5, 5.5
          2]]

Out[990]= {y → 3.27772}

Out[991]= {y → 5.61257}

Out[992]= {y → 5.46744}
```

Determines the likelihood of the first change being sucessful

```
In[993]:= ClearAll[y, z];
H = {{HA}, {HA}, {HB}, {HA}, {HD}};
EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
z = y - USLU;
AllCost =
 {TotCost[[1]], TotCost[[2]], TotCost[[3]], FailureCo:
ProbOK = {PDFChangeDist[[3]],
  PDFChangeDist[[4]], PDFChangeDist[[5]], {1}};
MinFun = {1, 1, 1, 1};
MinFun[[1]] = (Sign[BestCost - TotCost[[1]]] + 1);
MinFun[[2]] = (Sign[BestCost - TotCost[[2]]] + 1);
MinFun[[3]] = (Sign[BestCost - TotCost[[3]]] + 1);
MinFun[[4]] = (Sign[BestCost - FailureCost] + 1);
PFirstChange = MinFun * ProbOK;
AnyFirst = PFirstChange[[1, 1]] + PFirstChange[[2, 1]] +
  PFirstChange[[3, 1]] + PFirstChange[[4, 1]];
Plot[AnyFirst * UnitStep[z], {y, 2, 6}, PlotRange → All]
```




```
Out[1006]= - Graphics -
```

Integrates the minimum cost over the possible defects including the probability density function of the defect occurring.

```
In[1007]:= ClearAll[y, z];
           H = {{HA}, {HA}, {HB}, {HA}, {HD}};
           EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
           z = y - USLU;
           CEChange =
            NIntegrate[BestCost * CondDefDist, {y, USLU, Infinity

Out[1011]= 2.26473
```

The Expected cost is then simply the sum of the expected change cost and the unchanged cost and their occurrence.

```
In[1012]:= ClearAll[y, z];
           H = {{HA}, {HA}, {HB}, {HA}, {HD}};
           EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
           z = y - USLU;
           ExpectedCost = (Cost[[1]] * PDefect[[1, 1]] +
               CEChange * PDefect[[2, 1]])[[1]]
           ClearAll[H, EMod];

Out[1016]= 2.14528
```

Gives an overview report of the flexible design evaluation.

```
In[1018]:= H = {{HA}, {HA}, {HB}, {HA}, {HD}};
           EMod = {{EA}, {EA}, {EA}, {EC}, {ED}};
           FlexDesReport = Table[0, {i, 1, 6}, {j, 1, 2}];
           VIC = {{"Expected Cost", "H", "EMod",
              "P(NO Change)", "P(Any Change)", "USLU"}};
           FlexDesReport[[All, 1]] = VIC[[1, All]];
           FlexDesReport[[1, 2]] = ExpectedCost;
           FlexDesReport[[2, 2]] = HA;
           FlexDesReport[[3, 2]] = EA;
           FlexDesReport[[4, 2]] = PDefect[[1, 1]];
           FlexDesReport[[5, 2]] = PDefect[[2, 1]];
           FlexDesReport[[6, 2]] = USLU;
           FlexDesReport = SetPrecision[FlexDesReport, 4];
           MatrixForm[SetPrecision[FlexDesReport, 4]]
           ClearAll[H, EMod];
```

Out[1030]//MatrixForm=

Expected Cost	2.145
H	34.8
EMod	1.822×10^5
P(NO Change)	0.1927
P(Any Change)	0.807
USLU	2.899

Optimal Results

Uncertainty of RSM Model: Initial Design with least Expected Cost

Expected Cost	1.718
H	39.3
EMod	1.620×10^5
P(NO Change)	0.670
P(Any Change)	0.330
USLU	2.917

Uncertainty of RSM Model: Initial Design with least Part Cost (Robust Design)

Expected Cost	2.145
H	34.8
EMod	1.822×10^5
P(NO Change)	0.1927
P(Any Change)	0.807
USLU	2.899

APPENDIX C

INTERPOLATION AND EXTRAPOLATION

Some certain cases of assumptions are described below in more detail, as they are used frequently in engineering to predict the behavior of the design space based on a finite number of sample points. These methods are the interpolation between data and the extrapolation outside of existing data. These methods use simplifications and assumptions, however due to the frequent occurrence of interpolations and extrapolations in engineering design they are discussed in more detail below.

C.1 Interpolations

The prediction model often interpolates between known sample points by creating a response surface model. Theoretically, any continuous function can be used to interpolate between known sample points. First and second order functions are used frequently, creating a linear or quadratic interpolation between known sample points. This interpolation can contain both assumptions and simplifications. Depending on the interpolating function, the model might not fit the sample points correctly. If the number of sample points exceeds the number of model parameters, a perfect fit of the model to the data cannot be guaranteed nor is necessarily desirable. This is visualized in Figure 47, where four data points are utilized to determine three parameters of a quadratic equation.

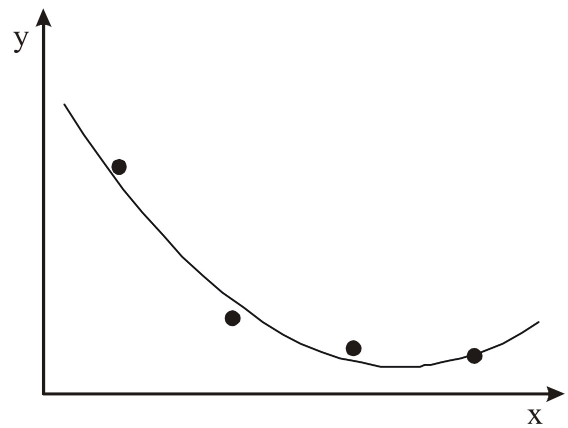


Figure 47: Simplifications during Model Interpolation

The actual behavior of the system between the sample points is usually not known, but assumed to behave according to the interpolating model. This is visualized in Figure 48, where an inverse function is interpolated using a quadratic function.

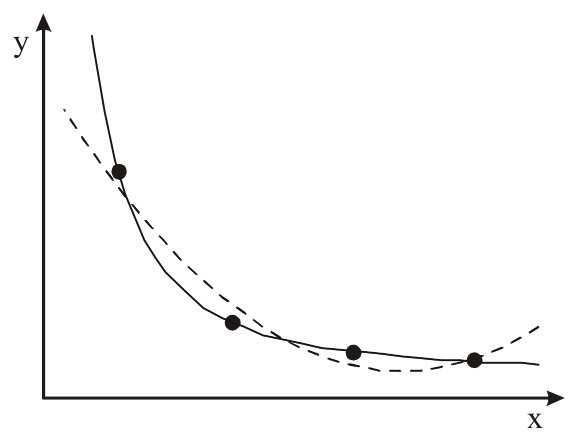


Figure 48: Interpolation Model Assumptions

Interpolations are a basis of design of experiments, where a linear or quadratic prediction function is usually fit to a number of sample points taken at specific locations within the design space. Fourier transformations also fit a function on existing data and are used frequently in frequency analysis to create a mathematical model of the measured frequency. The uncertainty due to interpolation has been investigated by numerous researchers. (Glaeser et al. 1995) analyzes mathematical techniques for uncertainty and sensitivity analysis, concluding that if an actual model is available it should be used and not simplified by fitting a prediction equation on the model due to the increased uncertainty. (Brown et al. 1998) determines the uncertainty of regression models based on uncertainty and variation in the sample data. (Chipman 1998) uses Bayesian methods to analyze and describe the model uncertainty of design of experiments. (Steele et al. 1993) sought to estimate the uncertainty of experiments with a small sample size, where for example only a single data value was measured, based on previous experience with similar cases.

C.2 Extrapolations

Extrapolations utilize the predicted model outside of the space investigated with the sample data. The possibility for error increases with the distance from the known data points. In general, an interpolation is preferred over an extrapolation due to the potential better model accuracy. However, in some cases it is only possible to take sample data on one side of the range of interest due to the complete unavailability or other extreme difficulty obtaining the data. The prediction accuracy of extrapolations can be improved by utilizing historic data of known cases. Figure 49 shows the example inverse function with a quadratic model extrapolated to about twice the width of the original data. The prediction becomes very inaccurate with increasing distance from the closest data point. In this case, it is not possible to make a valid extrapolation without the understanding of the basic behavior of the model. If for example it would be known that the original function has an inverse relation, the extrapolated function would also utilize an inverse term, extending the prediction accuracy over a greater range.

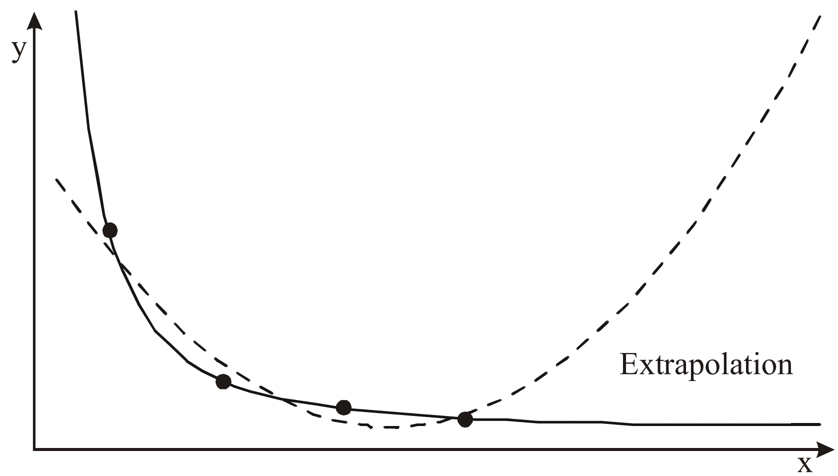


Figure 49: Extrapolation of Data with Large Error

A prime example of extrapolation is the behavior of a global system with time, e.g. the stock market or the weather report. The prediction of the weather of the next day would be much more accurate if the weather in two days would be known. However, the weather in two days is not (yet) known. Therefore, tomorrow's weather is extrapolated based on the known weather data. The prediction accuracy also decreases with the distance of the prediction. While the prediction of tomorrow's weather is usually fairly accurate, the prediction of the weather a week from today is much more uncertain. In engineering, extrapolations are frequently used in experiments, where a full-scale experiment would require too many resources. The aerodynamic behavior of airplanes is usually tested in a small scale and extrapolated to the aerodynamics of a full sized plane. Similarly, the flow dynamics of an ocean freighter is determined on a small scale and extrapolated to the predicted behavior of the full-scale ship. A comparison of the extrapolation model to historic cases is used to improve the accuracy of the prediction. (Cho et al. 1999) describes a method to reduce uncertainty in extrapolating small-scale model data using the example of a small ship model pulled through the water.

APPENDIX D

MONITOR HOUSING PREDICTION MODELS

D.1 Introduction

The prediction models for the monitor housing are a combination of response surface models and functional relations derived from analytical models and experimental data. The functional relations are identical for all material types, are listed below, and are based on the design variables and the response surface models. The response surface models differ for each material. The elements of the quadratic response surface are listed in the followings sections.

D.2 Design Variables

The utilized design variables are shown below, including the constraint limits spanning the valid design space.

Design Variable	Type	Unit	LCL	UCL
Melt Temperature	Continuous	ºC	230	270
Mold Temperature	Continuous	ºC	50	70
Eject Temperature	Continuous	ºC	60	90
Injection Time	Continuous	s	1	5
Thickness	Continuous	in	1.5	3.5
Flow Length	Continuous	cm	29	57
Material Type	Discrete	#	1	4
Number of Tools	Discrete	#	3	4
Availability	Continuous	hr/week	40	160
Projected Area	Continuous	cm^2	1,000	2,000
Production Volume	Discrete	#	10,000	1,000,000

D.3 Functional Relations

The tool cost increases linearly with the flow length from $150,000 for the lower constraint limit of the flow length at 29cm to $300,000 at the upper constraint limit of the flow length.

$$ToolCost = (FlowLength - 58) \cdot \frac{-37500}{7}$$

The tool cost per part depends on the number of tools, the production volume, and the tool cost.

$$ToolCostPerPart = \frac{NumberOfTools \cdot CurrentToolCost}{ProductionVolume}$$

The production time depends on the cycle time, the production volume, the availability, and the number of tools.

$$ProductionTime = \frac{CycleTime \cdot ProductionVolume}{Availability \cdot NumberOfTools \cdot 3600}$$

The clamp force depends on the melt pressure, the projected area, and the flow length.

$$ClampForce = \frac{(MeltPressure \cdot ProjectedArea) - \frac{2 \cdot MeltPressure}{3 \cdot FlowLength} \cdot \sqrt{\frac{Area^3}{\pi}}}{98.1}$$

The marginal part cost is the sum of the material cost, the process cost, and the tool cost per part.

$$MarginalPartCost = MaterialCost + ProcessCost + ToolCostPerPart$$

D.4 Response Surface Model Fitting Error

The table below gives an overview of the errors in fitting the quadratic response surface to the sample data for selected response predictions.

Response	% Fitting Error	Material 1	Material 2	Material 3	Material 4
Melt Pressure	Mean	-0.11	-0.21	-0.10	-0.9
	Deviation	9.05	10.72	9.54	10.89
Cycle Time	Mean	0.01	0.01	0.01	0.01
	Deviation	0.96	1.08	0.97	0.93
Shrinkage	Mean	0.00	0.00	0.00	0.00
	Deviation	0.00	0.00	0.00	0.00
Process Cost	Mean	0.03	0.02	0.03	0.03
	Deviation	2.10	2.18	2.11	2.08
Material Cost	Mean	0.00	0.00	0.00	0.00
	Deviation	0.00	0.00	0.00	0.00

D.5 Change Cost Matrix

The cost of a design failure is assumed to be $10 per part, representing the cost required for a redesign in order to satisfy the specifications.

	Mold Temperature	Thickness	#Tools	Material	Task Cost
Change Settings	0	1	1	0	
Remove Tool from Machine	0	1	0	0	120
Recut Tool	0	1	0	0	80000
Cut New Tool	0	0	1	0	200000
Insert Tool	0	1	1	0	120
Change Mold Temp	1	1	1	0	20
Remove Previous material	0	0	0	1	30
Dry material	0	0	0	1	10
Insert New material	0	0	0	1	20
Change Melt Temperature	0	0	0	1	25
Purge Material	0	0	0	1	25
Produce Part	1	1	1	1	20
Confirm Part Quality	1	1	1	1	60

D.6 Response Surface Model Material 1

	MeltPressure	CycleTime	Shrinkage	ProcessCost	MaterialCost
Constant	953.6630	-14.34062536	0.419999838	-1.1923832	1.46031E-06
MeltTemp	-2.3703	0.575199472	-3.07371E-16	0.0199722	-8.56953E-15
MoldTemp	-1.7126	-1.527865174	-0.007499994	-0.053050869	-2.22988E-14
EjectTemp	0.2901	-0.218913755	4.78495E-16	-0.007601165	1.55801E-14
InjectionTime	83.9043	1.686768758	2.57823E-16	0.058568368	9.29812E-15
FlowLength	13.4039	0.481942962	3.56623E-16	0.016734149	9.47488E-15
Thickness	-539.7894	4.165143997	-2.00708E-15	0.144623031	2.0778687
MeltTemp * MoldTemp	0.0029	0.000176947	0	6.14402E-06	0
MeltTemp * EjectTemp	0.0000	4.96705E-11	0	1.5522E-12	0
MeltTemp * Injectionime	-0.0295	3.72529E-10	0	-1.16415E-11	0
MeltTemp * FlowLength	-0.0139	0	0	0	0
MeltTemp * Thickness	0.4209	0.023013896	0	0.000799093	0
MoldTemp * EjectTemp	0.0000	-1.6888E-09	0	-7.76102E-11	0
MoldTemp * InjectionTime	-0.2704	-7.45058E-10	0	-1.62981E-10	0
MoldTemp * FlowLength	-0.0077	0	0	0	0
MoldTemp * ThicknessIn	0.6947	0.127413951	0	0.004424096	0
EjectTemp * InjectionTime	0.0000	-1.49012E-09	0	-1.70742E-10	0
EjectTemp * FlowLength	0.0000	0	0	0	0
EjectTemp * Thickness	0.0000	-0.20074216	0	-0.006970214	0
InjectionTime* FlowLength	0.3908	0	0	0	0
InjectionTime* Thickness	-22.4292	7.45058E-09	0	-1.16415E-09	0
FlowLength* Thickness	-2.1890	0	0	0	0
$MeltTemp^2$	0.0012	-0.001220246	6.14743E-19	-4.23696E-05	1.71391E-17
$MoldTemp^2$	-0.0038	0.011305169	5E-05	0.000392541	1.85824E-16
$EjectTemp^2$	-0.0019	0.002857905	-3.18996E-18	9.92328E-05	-1.03868E-16
$InjectionTime^2$	0.0237	-0.114461453	-4.2826E-17	-0.003974353	-1.55431E-15
$FlowLength^2$	-0.0142	-0.005073084	-3.76738E-18	-0.000176149	-9.97971E-17
$Thickness^2$	83.3348	1.957968586	3.98986E-16	0.067985039	3.57628E-07

D.7 Response Surface Model Material 2

	MeltPressure	CycleTime	Shrinkage	ProcessCost	MaterialCost
Constant	953.6630	-14.34062536	0.419999838	-1.1923832	1.46031E-06
MeltTemp	-2.916432	0.714025	-3.07E-16	0.024793	-9.9747E-15
MoldTemp	-1.500862	-1.896617	-0.0075	-0.065855	-2.3331E-14
EjectTemp	0.319798	-0.27175	4.78E-16	-0.009436	1.8624E-14
InjectionTime	64.27515	1.852521	2.58E-16	0.064324	8.9859E-15
FlowLength	13.2699	0.598261	3.57E-16	0.020773	1.204E-14
Thickness	-502.1715	5.170409	-2.01E-15	0.179528	2.2638011
MeltTemp * MoldTemp	0.003142	0.00022	0	7.63E-06	0
MeltTemp * EjectTemp	0	9.44E-10	0	2.95E-11	0
MeltTemp * Injectionime	-0.032539	3.73E-10	0	1.16E-11	0
MeltTemp * FlowLength	-0.015673	0	0	0	0
MeltTemp * Thickness	0.498297	0.028568	0	0.000992	0
MoldTemp * EjectTemp	0	1.89E-09	0	1.02E-10	0
MoldTemp * InjectionTime	-0.239172	7.45E-10	0	-1.16E-10	0
MoldTemp * FlowLength	-0.007562	0	0	0	0
MoldTemp * ThicknessIn	0.637781	0.158165	0	0.005492	0
EjectTemp * InjectionTime	0	4.97E-10	0	-3.26E-10	0
EjectTemp * FlowLength	0	0	0	0	0
EjectTemp * Thickness	0	-0.249191	0	-0.008652	0
InjectionTime* FlowLength	0.315609	0	0	0	0
InjectionTime* Thickness	-17.23047	-7.45E-09	0	3.03E-09	0
FlowLength* Thickness	-2.128256	0	0	0	0
$MeltTemp^2$	0.002163	-0.001515	6.15E-19	-5.26E-05	1.9949E-17
$MoldTemp^2$	-0.004747	0.014034	5E-05	0.000487	1.9443E-16
$EjectTemp^2$	-0.002132	0.003548	-3.19E-18	0.000123	-1.2416E-16
$InjectionTime^2$	0.356327	-0.142087	-4.28E-17	-0.004934	-1.5023E-15
$FlowLength^2$	-0.010024	-0.006297	-3.77E-18	-0.000219	-1.2655E-16
$Thickness^2$	73.27531	2.430526	3.99E-16	0.084393	2.3842E-07

D.8 Response Surface Model Material 3

	MeltPressure	CycleTime	Shrinkage	ProcessCost	MaterialCost
Constant	1061.506	-15.14566	0.42	-1.220337	2.1756E-06
MeltTemp	-2.976547	0.588683	-3.07E-16	0.02044	-8.3787E-15
MoldTemp	-1.922636	-1.563678	-0.0075	-0.054294	-2.1366E-14
EjectTemp	0.220819	-0.224045	4.78E-16	-0.007779	1.5319E-14
InjectionTime	90.10143	1.702867	2.58E-16	0.059127	5.676E-15
FlowLength	13.61516	0.49324	3.57E-16	0.017126	8.4085E-15
Thickness	-564.6116	4.262779	-2.01E-15	0.148013	1.91803241
MeltTemp * MoldTemp	0.002905	0.000181	0	6.29E-06	0
MeltTemp * EjectTemp	0	-2.48E-10	0	-1.24E-11	0
MeltTemp * Injectionime	-0.036055	3.73E-10	0	0	0
MeltTemp * FlowLength	-0.015281	0	0	0	0
MeltTemp * Thickness	0.480797	0.023553	0	0.000818	0
MoldTemp * EjectTemp	0	4.97E-10	0	5.59E-11	0
MoldTemp * InjectionTime	-0.292703	-7.45E-10	0	-4.66E-11	0
MoldTemp * FlowLength	-0.007016	0	0	0	0
MoldTemp * ThicknessIn	0.750281	0.130401	0	0.004528	0
EjectTemp * InjectionTime	0	4.97E-10	0	2.17E-10	0
EjectTemp * FlowLength	0	0	0	0	0
EjectTemp * Thickness	0	-0.205448	0	-0.007134	0
InjectionTime* FlowLength	0.384885	0	0	0	0
InjectionTime* Thickness	-23.33703	-7.45E-09	0	-4.19E-09	0
FlowLength* Thickness	-2.187401	0	0	0	0
$MeltTemp^2$	0.002209	-0.001249	6.15E-19	-4.34E-05	1.6757E-17
$MoldTemp^2$	-0.003262	0.01157	5E-05	0.000402	1.7805E-16
$EjectTemp^2$	-0.001472	0.002925	-3.19E-18	0.000102	-1.0214E-16
$InjectionTime^2$	0.074693	-0.117144	-4.28E-17	-0.004068	-9.4369E-16
$FlowLength^2$	-0.013088	-0.005192	-3.77E-18	-0.00018	-8.8726E-17
$Thickness^2$	84.91378	2.003864	3.99E-16	0.069579	3.5763E-07

D.9 Response Surface Model Material 4

	MeltPressure	CycleTime	Shrinkage	ProcessCost	MaterialCost
Constant	1069.545	-12.81427	0.42	-1.139384	2.3544E-06
MeltTemp	-3.458471	0.549633	-3.07E-16	0.019084	-1.1588E-14
MoldTemp	-1.827734	-1.459955	-0.0075	-0.050693	-2.1133E-14
EjectTemp	0.261228	-0.209184	4.78E-16	-0.007263	1.6193E-14
InjectionTime	91.83537	1.656244	2.58E-16	0.057508	6.7446E-15
FlowLength	13.02322	0.460523	3.57E-16	0.01599	9.7641E-15
Thickness	-536.316	3.980016	-2.01E-15	0.138195	1.99115396
MeltTemp * MoldTemp	0.002877	0.000169	0	5.87E-06	0
MeltTemp * EjectTemp	0	-2.98E-10	0	-4.66E-12	0
MeltTemp * Injectionime	-0.035852	7.45E-10	0	5.82E-11	0
MeltTemp * FlowLength	-0.014442	0	0	0	0
MeltTemp * Thickness	0.449953	0.021991	0	0.000764	0
MoldTemp * EjectTemp	0	-2.78E-09	0	-1.15E-10	0
MoldTemp * InjectionTime	-0.293891	0	0	-2.33E-11	0
MoldTemp * FlowLength	-0.007424	0	0	0	0
MoldTemp * ThicknessIn	0.755156	0.121751	0	0.004227	0
EjectTemp * InjectionTime	0	0	0	-2.02E-10	0
EjectTemp * FlowLength	0	0	0	0	0
EjectTemp * Thickness	0	-0.19182	0	-0.00666	0
InjectionTime* FlowLength	0.403306	0	0	0	0
InjectionTime* Thickness	-23.52266	0	0	2.56E-09	0
FlowLength* Thickness	-2.125033	0	0	0	0
$MeltTemp^2$	0.003383	-0.001166	6.15E-19	-4.05E-05	2.3176E-17
$MoldTemp^2$	-0.003918	0.010803	5E-05	0.000375	1.7611E-16
$EjectTemp^2$	-0.001742	0.002731	-3.19E-18	9.48E-05	-1.0794E-16
$InjectionTime^2$	-0.13171	-0.109374	-4.28E-17	-0.003798	-1.1172E-15
$FlowLength^2$	-0.013095	-0.004848	-3.77E-18	-0.000168	-1.0256E-16
$Thickness^2$	81.64316	1.870942	3.99E-16	0.064963	5.9605E-07

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