Definition and Application of a Process Flexibility Index

David Kazmer

David Hatch

Liang Zhu

Christoph Roser

Deepak Kapoor

Department of Mech. & Ind. Engineering,
University of Massachusetts Amherst,
Amherst, MA 01003

A performance index, C_f, is developed for assessing the flexibility of candidate engineering designs and manufacturing processes at various stages during product development. The index is a ratio of the range of the feasible space to the range of the specification or measured process variation. The process flexibility index complements the process capability index, C_p, to provide a simple and quantitative basis for practicing the principles of axiomatic design. This quantitative measure can be used to support rational product development and manufacturing investment decisions, and clearly illustrates important concepts regarding controllability across conflicting specifications. A high C_p indicates that the design or process can consistently manufacture the product within precisely defined performance specifications. A high C_f indicates that the process can be easily changed to meet diverse performance specifications. The evaluation of C_f requires the mapping of the global feasible space, the solution to which is also discussed.
[DOI: 10.1115/1.1536174]

1 Introduction

Continued global competitiveness has increased standards for product quality and performance while requiring reduced product development time and unit cost. In this new manufacturing paradigm, the efficient development and delivery of robust product and process designs differentiates the market leader from the follower. Consider the following trends in the product development cycle: reduction of typical product development times by 10% per year [1]; increasing mass customization and proliferation of product variety [2]; increasing reliance on electronic design and manufacturing environments [3]; continued global dispersion of the product development team [4]; and continued horizontal outsourcing of production. It is not uncommon for 80% of a company's revenue to be derived from products that are less than two years old [5].

For the design engineer, these trends increase the pressure to rapidly develop functional designs that can be easily manufactured. In theory, improved design and communication tools should enable the designer to more accurately define the product specifications, more quickly synthesize robust design candidates, and more accurately evaluate the design performance and manufacturability. In reality, increased product variety and reduced product development time reduces the allowable engineering design time while the topology of the globally distributed product development team obscures the design interfaces and inhibits communication with manufacturing.

For the process engineer, these trends place enormous pressure to deliver functional products at the lowest possible cost. In theory, robust designs will be electronically transmitted through a virtual, agile manufacturing network to be efficiently manufactured and distributed [6]. From a fundamental perspective, however manufacturing processes have not improved at a rate equivalent to advancing manufacturing requirements. While some advances in set-up and control have certainly been made, the processes have not become significantly more capable or flexible. As a result, engineering designs and manufacturing processes must frequently be altered to obtain the desired product performance and/or acceptable quality levels. The lack of robustness in the integrated product and process development cycle is sometimes evidenced by long product development cycles, excessive tooling costs, low process yields, and unacceptable product quality. As is well known, engineering changes become more difficult and expensive to correct as the product development progresses [7]. Dacey [8] has quantified that late design iterations often require costly tooling changes and delay the product launch.

This paper more rigorously develops the process flexibility index which was first developed to evaluate the ability of a manufacturing process to deliver a broad set of product attributes relative to the product specifications [9]. Process flexibility is desirable in that it ensures that errors in the design, unaccounted customer requirements, and significant sources of variation can be compensated in the manufacturing process without product redesign. This product and process design freedom is vital to the successful development and timely delivery of quality products to the marketplace.

In the next section, a method for mapping the feasible space from approximate response surfaces is described. The feasible space is then utilized in defining the process flexibility metric for applications with one and two-sided specifications. Finally, the process flexibility metric is applied to two real manufacturing case studies.

2 Feasibility Mapping

The first objective of most manufacturing development processes is to generate a feasible product that satisfies specified quality requirements. Based on the complexity and capability of the process, a preferred trade-off is then made between multiple quality attributes, manufacturing cost, and set-up time. Unfortunately, multiple quality attributes and manufacturing cost are tightly coupled in most applications, increasing the difficulty of achieving an "optimal" manufacturing set-up.

Algorithms have been recently developed to map the process feasibility given a system model, thereby providing an estimate of the global process window [10,11]. The feasibility map, or process window, can be generated from a set of specifications and predictive models. The models, which relate controlled product and process variables to product quality attributes, may be derived from analysis, experience, and/or experiments utilizing design of experiment or response surface methods. These prediction models typically have the form:

$$y_i = f_i(x_1, x_2, x_3, \ldots, x_n) \tag{1}$$

Denoting the ith quality attribute as y_i, a typical specification can be expressed as $LSL_i \leq y_i \leq USL_i$, where LSL and USL represent the lower and upper specification limit. In many cases, the quality attribute is a one-sided constraint, defined as $LSL_i \leq y_i$ or y_i

Contributed by the Manufacturing Engineering Division for publication in the JOURNAL OF MANUFACTURING SCIENCE AND ENGINEERING. Manuscript received August 2000; Revised November 2001. Associate Editor: J. Hu.

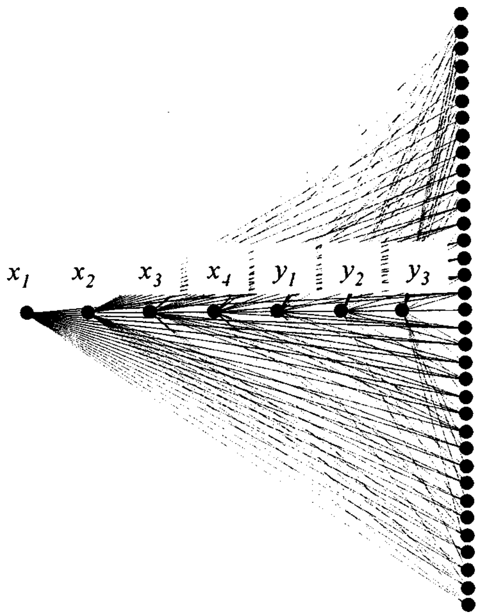

Fig. 1 k-subset of four control variables and three quality attributes

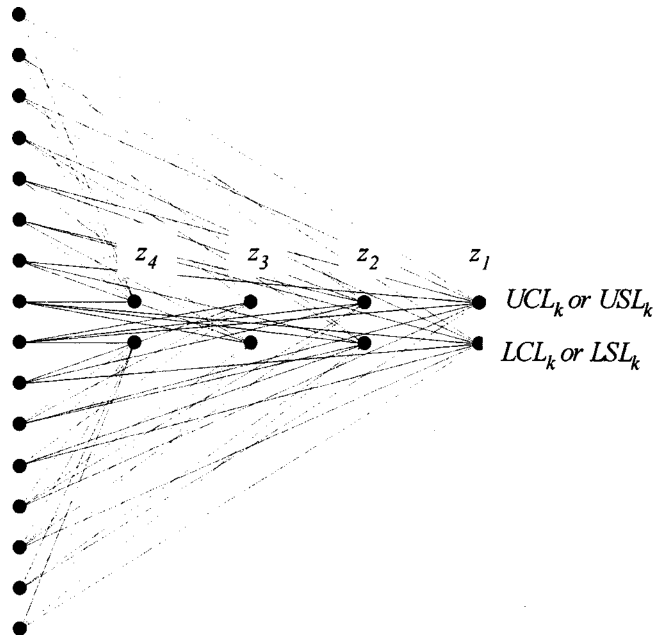

Fig. 2 Permutation of constraints for k-subset with four variables

$\leqslant USL_i$. As a whole, it is not trivial to define the proper specification limits due to the potential conflict among the multiple quality attributes. Working with the development team, the product and process engineer may jointly update the specification limits during an iterative development process.

The details of the algorithm for mapping the product/process feasibility are available [10,11]. Essentially, the algorithm is a three-step process: 1) combinatorial sub-system generation; 2) analysis of permuted constraints; and 3) resolution of extreme points. Each of these steps will be briefly described. An improved extensive simplex method has also been developed for efficiently mapping the feasible space [12].

2.1 Combinatorial Sub-System Generation. Consider a system consisting of *n* design or process variables and *m* quality attributes. The extreme limits of the feasible process are formed by the boundary of one or more active constraints in the defined system. The key to finding feasibility boundary then, is to establish the critical constraint combinations for the system. The first step in the solution requires the generation of a list *z*, composed of *m* elements of *x* augmented with *n* elements of *y*:

$$z=\{x_1,x_j,\cdots,x_m,y_1,y_i,\ldots,y_n\} \tag{2}$$

A k-subset [13] is then defined consisting of exactly C^n_{n+m} elements, where

$$C^n_{n+m}=\frac{(n+m)!}{n!m!} \tag{3}$$

This set represents all possible combinations of the control variables, *x*, with the quality attributes, *y*. For four control variables and three quality attributes, a set of thirty-five sub-systems needs to be analyzed. For this example, Fig. 1 shows a graph representing the 35 subset combinations, where $\{C_1\}=\{x_1,x_2,x_3,x_4\}$, $\{C_2\}=\{x_1,x_2,x_3,y_1\}$, and $\{C_{35}\}=\{x_4,y_1,y_2,y_3\}$.

2.2 Analysis of Permuted Constraints. Each defined subsystem consisting of *m* variables is then analyzed for all permutations of the defined constraints. The permutation list consists of P^2_m elements where

$$P^2_m=2^m \tag{4}$$

This set represents the control limits and specification limits that correspond to the variables defined in the C_k subset. For the given example with four control variables, Fig. 2 shows a graph of sixteen constraints, where for $P_1|C_1=\{LCL_1,LCL_2,LCL_3,LCL_4\}$, $P_2|C_1=\{LCL_1,LCL_2,LCL_3,UCL_4\},\ldots,P_{15}|C_{35}=\{LCL_4,USL_1,USL_2,USL_3\}$, $P_{16}|C_{35}=\{UCL_4,USL_1,USL_2,USL_3\}$.

A set of extreme points is then solved for each k-subset of the system with its corresponding permutation of constraints. The number of sub-system analyses that are performed is equal to $C^m_{n+m}\cdot P^2_m$, or 35 times 16 equals 560 cases for the preceding example as represented by the graph shown in Fig. 3. It should be noted that only a small number of the sub-systems that are ana-

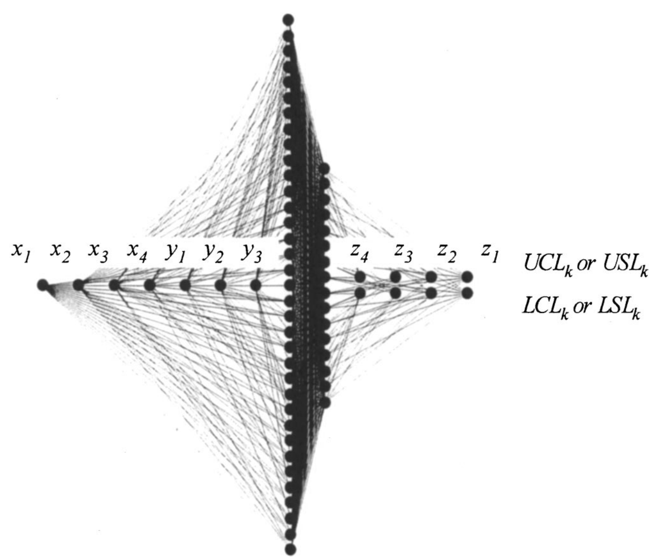

Fig. 3 Graph of k-subset combinations and constraint permutations

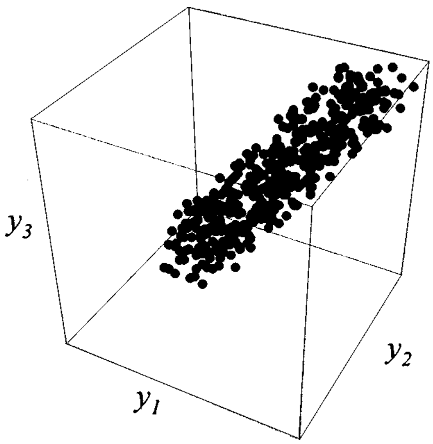

Fig. 4 Extreme points from all subsystems

lyzed will satisfy the feasibility requirements of all process variables and quality attributes, and an even smaller set of the feasible points will be global extrema.

2.3 Resolution of Extreme Points. Process engineers are interested in the global feasible set of process conditions and quality attributes. The result of the preceding analysis is an exhaustive set of processing conditions and their resulting quality attributes, where each set corresponds to the extreme points of a derived subsystem. Only a subset of these points lay on the extreme feasible boundary of the global system. Continuing with the previous example, Fig. 4 plots the quality attributes of the set of 560 points corresponding to the solution of all k-subset combinations and constraint permutations.

For linear system models, convexity properties significantly simplify the solution of the global feasible space [14]. Based on the convexity, the global feasible space is the convex hull of the extreme points. Convex hull methods, such as qHull [15], enable the efficient reduction of potential extrema to derive the desired feasibility maps. Figure 5 plots the global feasible space for the preceding example.

The set of extreme points can be utilized for vital process interpretation by inspecting the graph of active constraint sets. Figure 6 is a graph of the k-subset combinations and constraint permutations from Fig. 3 where all nonextreme subsets have been removed. The set has been reduced from 560 subsystems to 16 subsystems. Though difficult to inspect the static image, the graph representation provides vital information about the trade-off between process variable and multiple quality attributes. Figure 6,

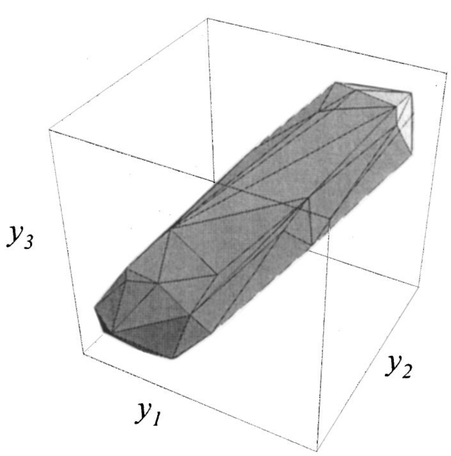

Fig. 5 Global extreme points for system

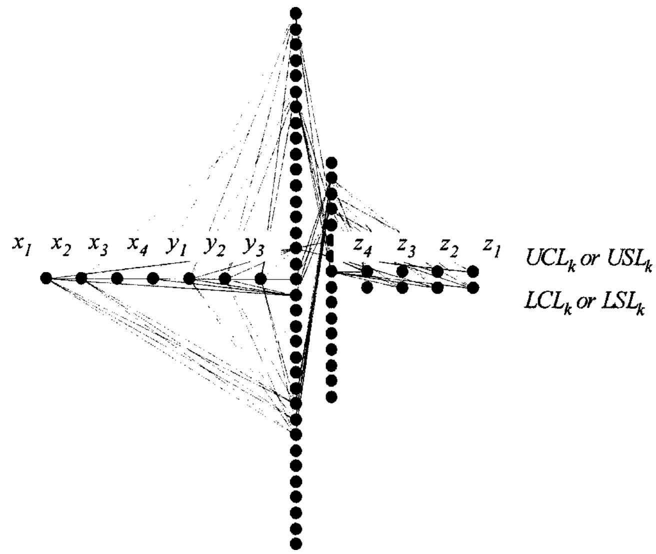

Fig. 6 Graph of dominating constraints

for example, indicates that x_3 is never critical to changing the quality attributes. Detailed inspection also indicates that the lower specification limits on y_2 and y_3 are never critical, since no connections are made to the nodes on the lower right hand corner of Fig. 6. Such information is useful in specification development, process design, and process set-up.

2.4 Discussion. The feasible space provides quantitative bounds on process variables and quality attributes based on system models. The feasibility space allows the process engineer to explicitly examine the trade-off between multiple quality attributes, or to compensate for a change in a processing variable by changing other processing variables. Alternatively, the process engineer may decide to hold certain processing variables and/or quality attributes fixed, and selectively examine the effect of remaining processing variables on the possibly reduced range of quality attributes.

Like most practical problems, the exploration of the global feasible space is a high-order polynomial or NP problem. The C^n_{n+m} constraint combinations dominate the polynomial order of the calculation time. However, a lower-upper decomposition [16] adopted in the algorithm has decreased the number of the linear system equations from C^n_{2n+2m} to C^n_{n+m}. Moreover, the left hand side corresponding to the coefficients of the *k*th subsystem only needs to be inverted once for each set of permuted constraints. When the number of quality attributes is under 10, the solution requires negligible computation time. Numerical methods, such as Monte-Carlo simulation, have been implemented for validation purposes and shown to require orders of magnitude greater computational time at reduced accuracy.

3 Definition of the Process Flexibility Metric

The quality of products and efficiency of processes are frequently evaluated by various statistical tools based on distributions and control charts. A common estimate of the ability of the process to produce consistent product relative to specification is provided by the process capability index, defined as:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{5}$$

where *USL* and *LSL* represent the upper and lower specification limits of a quality attribute, and σ is the measured standard deviation of the quality attribute. This definition holds true only if the

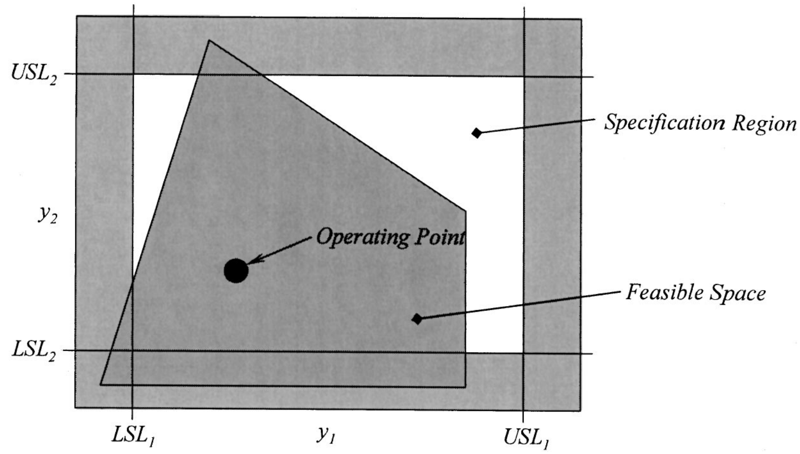

Fig. 7 Feasible space, specifications, and operating point for two quality attributes with upper and lower specification limits

manufacturing process is centered within the specification limits. If the nominal is elsewhere, another index, C_{pk} is used:

$$C_{pk}=\min\left[\frac{\mu-LSL}{3\sigma},\frac{USL-\mu}{3\sigma}\right] \tag{6}$$

where μ represents the mean of the manufactured attribute. C_{pk} provides an indication of the relative proximity of the quality attribute to the specification. Additional types of process capability indices have been reviewed [17].

Maintaining a high value of C_p is desirable. The consistency, governed by a low standard deviation, is indicative of the process robustness—insensitivity to uncontrolled variation. However, satisfactory process capability indices do not necessarily guarantee production success. In fact, production delays are more typically due to unforeseen product requirements and poor deterministic assumptions rather than issues surrounding manufacturing variation.

To facilitate product and process development, the process flexibility metric, C_f, is defined as a ratio of the likelihood of operating the process within its feasible region to the likelihood of operating the process within the specification limits.

$$C_f=\frac{\text{Size of Feasible Space}}{\text{Size of Specification Region}} \tag{7}$$

As the process flexibility index increases, the manufacturing process is increasingly able to significantly alter the product quality attributes relative to the product specifications. The process flexibility index complements the process capability indices, C_p and C_{pk}, to provide a simple and quantitative basis for comparison of candidate manufacturing processes.

3.1 Uniformly Distributed Quality Attributes. Figure 7 illustrates an example with two quality attributes operating at the point indicated. The quality attributes both have a two-sided specification and together form a rectangular area that is acceptable in the center of the figure. The feasible space is indicated by the shaded trapezoid, which cannot be increased in size without violating other quality attributes or exceeding the control limits of the processing variables.

Suppose that the likelihood of setting a quality attribute to any specific value is uniformly distributed with a probability p. In this case, the likelihood, ϕ, of accepting a set of quality attributes within the specification range is:

$$\phi^{specification\_range}=p\cdot V^{specification\_range} \tag{8}$$

where $V^{specification\_range}$ indicates the volume of the specifications, which is computed as a simple multiplication of the specification distances. Similarly, the likelihood of accepting a set of quality attributes within the global feasible space is:

$$\phi^{feasible\_range}=p\cdot V^{feasible\_range} \tag{9}$$

where the volume of the feasible space, $V^{feasible\_range}$, can be readily computed using quick hull algorithms [15]. The process flexibility index can then be computed as a ratio of the two probabilities:

$$C_f=\frac{\phi^{feasible\_range}}{\phi^{specification\_range}}=\frac{p\cdot V^{feasible\_range}}{p\cdot V^{specification\_range}}=\frac{V^{feasible\_range}}{V^{specification\_range}} \tag{10}$$

For a uniform distribution, the process flexibility index is simple a ratio of the size of the feasible space to the size of the specification region. As such, this definition is useful when preference to quality attributes is unknown. A C_f greater than 1.0 indicates that the process is able to deliver a more diverse set of quality attributes than those defined in the specifications. This definition of the process flexibility index is not dependent on the current operating point, since the likelihood of a change is uniformly distributed.

Many quality attributes, however, are defined with a one-sided specification. For example, part weight may have a specified maximum, or a manufactured material property may have a specified minimum. The described algorithms for computing the feasible space continue to function for such one-sided specifications; reducing the number of specifications will simply tend to increase the feasible space. However, this definition of the process flexibility metric is not valid for one-sided specifications, since the volume of the specification range is undefined when there is no bound for the complementary specification limit. One approach for one-sided specifications is to use a weighting function to discount the likelihood of a quality attribute occurring very far from the operating region. This approach leads directly to reasonable probabilistic approach, which is next discussed.

3.2 Normally Distributed Quality Attributes. The selection of the weighting function is of primary importance in the evaluation of needed process flexibility. Ideally, the weighting function should represent the true likelihood of process selection based on the necessary changes in the quality attributes. To accommodate inevitable shifts of the average in a process, Motorola Six Sigma guidelines have specified a $\pm6\sigma$ production tolerance on either side of the nominal value [18]. It is commonly believed that the 6σ approach was developed to reduce the defect rate to

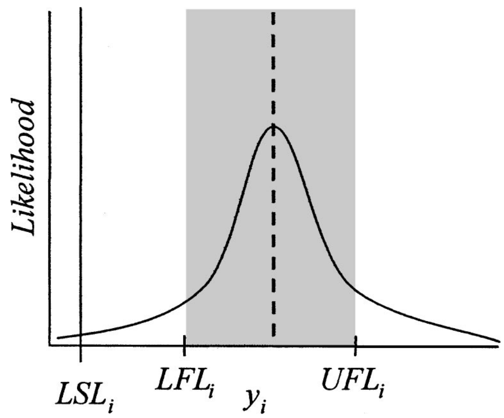

Fig. 8 Three approaches to weighting function definition

less than three rejections per million manufactured units. However, the original motivation of the approach was to ensure a 4.5σ yield given a long-term $\pm 1.5\sigma$ shift in the mean or specifications! Based on this premise, a probabilistic approach is developed that explicitly defines the flexibility of the process.

Consider the probability density function about a current operating point as shown in Fig. 8. This density function does not represent the distribution of the *i*th quality attribute due to noise, but rather the likelihood for needing to change the *i*th quality attribute due to external effects. As shown in the figure, there is equal probability that the quality attribute may need to be increased or decreased from the mean value, and some small probability that the quality attribute will need to be reduced below the lower specification limit.

The shaded area indicates the feasible region for this quality attribute without violating other quality attributes or requiring the process to operate beyond its specified control limits. Using normal statistics, the probability of the process being able to accommodate changes in the *i*th quality attribute according to the prescribed probability density function is:

$$\phi_i^{feasible\_range}=\int_{LFL_i}^{UFL_i} pdf(y_i)dy_i \tag{11}$$

where LFL_i and UFL_i are the lower and upper feasible limits for the *i*th quality attribute, as derived from the described feasibility analysis. The likelihood for evaluating the need for a process change within the specified limits can be similarly evaluated as:

$$\phi_i^{specification\_range}=\int_{LSL_i}^{USL_i} pdf(y_i)dy_i \tag{12}$$

where LSL_i and USL_i are the specification limits for the *i*th quality attribute. The complementary specification limit may be set to $-\infty$ or $+\infty$ for one-sided specifications. The univariate process flexibility index for both one and two sided specifications is then defined as:

$$C_f^i=\frac{\Phi^{-1}(\phi_i^{feasible\_range})}{\Phi^{-1}(\phi_i^{specification\_range})} \tag{13}$$

where Φ^{-1} is the inverse of the standard normal cumulative distribution. This definition of the process flexibility is dependent upon the current operating point of the manufacturing process, since this determines the likelihood of operating outside the specification or feasible range. One common simplification that can be made is to compare the breadth of the feasible range directly to the variation of the process:

$$C_f^i=\frac{UFL-LFL}{6\sigma} \tag{14}$$

A process flexibility index can be computed for each quality attribute. Since most manufacturing processes must meet multiple quality specifications, it is desirable to condense this vector of flexibility indices down to one scalar that is representative of the composite process flexibility. Several forms of multivariate indices have been proposed for process capability [19] and product robustness [20]. The multivariate index of process flexibility should reflect the combined loss if the manufacturing flexibility is insufficient to deliver multiple quality attributes. The joint probability of the process delivering needed changes in the quality attributes within the feasible region is:

$$\phi^{feasible\_range}=\prod_{i=1}^{n} \phi_i^{feasible\_range} \tag{15}$$

The joint probability of the process delivering needed changes within defined specifications is:

$$\phi^{specification\_range}=\prod_{i=1}^{n} \phi_i^{specification\_range} \tag{16}$$

The multivariate process flexibility index may then be defined as:

$$C_f=\frac{\Phi^{-1}(\phi^{feasible\_range})}{\Phi^{-1}(\phi^{specification\_range})} \tag{17}$$

4 Two Applications of the Process Flexibility Metric

4.1 A New Injection Molding Process. To investigate the flexibility of the injection molding process, a half-factorial design of experiments [21] was performed to determine the main effects between the important process parameters and three critical part dimensions in a commercial printer housing:

$$\begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix}=\begin{bmatrix} 0.57 & -0.10 & 0.43 & 0.02 \\ 0.51 & -0.18 & 0.29 & 0.00 \\ 0.23 & -0.05 & 0.18 & 0.10 \end{bmatrix}\begin{bmatrix} Pressure \\ Velocity \\ Temperature \\ ScrewSpeed \end{bmatrix} \tag{18}$$

In this equation, the machine parameters have been scaled to the range of 0 to 1, indicative of the maximum feasible processing range for this application. The resulting coefficients of the linear model are actual changes in part dimensions (measured in mm). It should be noted that once tooling is completed, the dimensional changes available through processing are quite limited though functionally significant.

There are two significant conclusions that can be drawn from this system model. First, all three of the dimensions react similarly to changes in the process settings. Thus, the molding process is nearly fully coupled and behaves as a one degree of freedom process in which only one quality attribute is controllable. Second, the equation shows the relative effect that each of the processing variables can have on the product quality attributes. Pressure was the most significant process variable, followed by temperature, velocity, and others.

To enhance the flexibility of the molding process, dynamic valves were designed and implemented to meter the flow and pressure of the melt to the mold cavity [22]. The current implementation, Dynamic Feed®, is shown in Fig. 9. The pressure drop and flow rate of the melt is dynamically varied by the axial movement of a valve which controls the gap between the valve and the mold wall. By de-coupling the control of the melt for each valve, melt control at each gate can override the molding machine settings and provide better time response and differential control of the melt. Each valve acts as an individual injection unit, lessening dependency on machine dynamics.

The material shrinkage and dimensions change at differing locations in the part based on the pressure contours in the mold cavity. The ability to change individual dimensions or other quality attributes without re-tooling mold steel provides significant

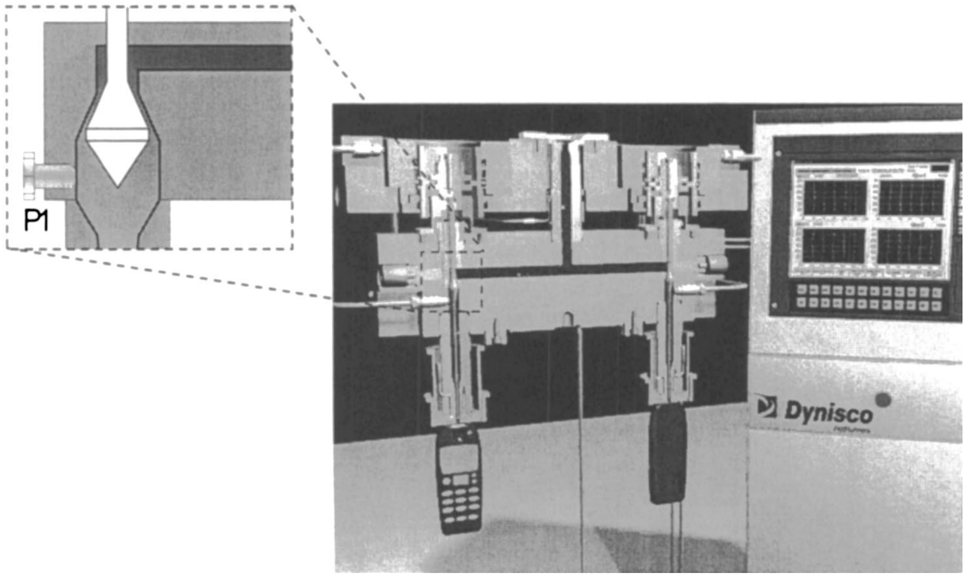

Fig. 9 Dynisco HotRunner's Dynamic Feed® System

process flexibility. It is possible to augment the system model with the additional degrees of freedom provided by the multiple command pressures, $P_{i\in[1,4]}$, and re-examine the controllability of the three part dimensions:

$$\begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix} = \begin{bmatrix} -0.02 & -0.05 & 0.08 & -0.01 \\ -0.03 & -0.09 & 0.05 & 0.00 \\ -0.01 & -0.02 & 0.03 & 0.01 \end{bmatrix} \begin{bmatrix} Pressure \\ Velocity \\ Temperature \\ ScrewSpeed \end{bmatrix} + \begin{bmatrix} 0.00 & 0.31 & 0.60 & 0.00 \\ 0.10 & 0.17 & 0.00 & 0.16 \\ 0.00 & 0.02 & 0.00 & 0.21 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \quad (19)$$

There are two significant implications of this result. First, the closed loop control of cavity pressures has significantly reduced the dependence of part dimensions on machine settings, as indicated by the reduction in the magnitude of coefficients for the primary machine settings. This effect has also been evidenced by reductions in the measured standard deviations of multiple part dimensions in multiple applications, typically increasing the process capability index, C_p, from less than 1 to greater than 2.

Utilizing the described methods, the feasible space for the conventional and new molding process are mapped in Fig. 10. The process flexibility for conventional molding was computed for a uniform distribution of part dimensions as 0.02, barely capable producing of any changes in product quality. The lack of process flexibility is a primary reason for engineering changes and delays for molded components. The process flexibility for Dynamic Feed® was similarly calculated as 0.7. The greater flexibility of the new process enabled four companies to cut the time from mold design to finished part down to hours instead of weeks, as validated for five different applications in five successive days at National Plastics Exposition in the McCormick Center in Chicago [23].

4.2 DVD Processing. Injection molding is also the primary manufacturing process in the creation of compact and digital video discs. Each disc is composed of an optically transparent substrate (typically polycarbonate), with one or more substrates containing a reflective metalized data surface. For prerecorded media, the data is stored on a disc in the form of pits that are molded into the disc during the injection molding process. The data is part of the disc; the data is not written in a secondary operation as in magnetic media. For DVD media, two 0.6 mm substrates are bonded together to increase data storage capacity. The bonding process combined with the small definition of data pits requires stringent flatness specifications of each DVD substrate. In addition, substrate thickness and birefringence play significant roles in the ability of the DVD laser to properly read the

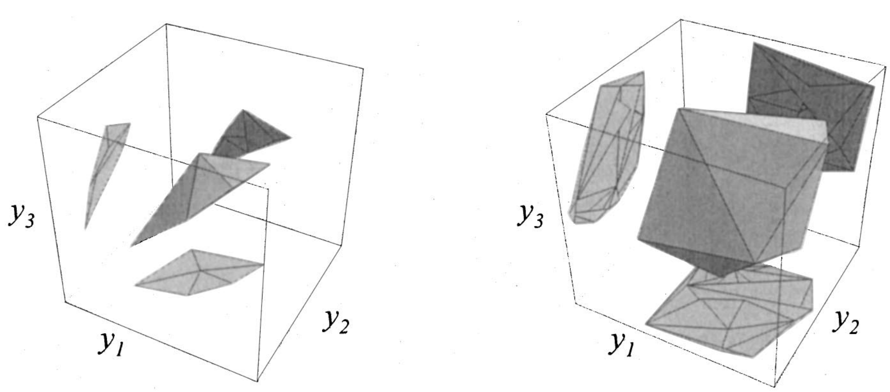

Fig. 10 Feasible performance spaces for conventional molding and Dynamic Feed®

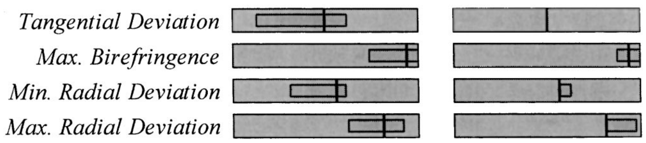

Fig. 11 Feasible performance limits for initial and restricted DVD manufacturing process

optical media [24–26]. The number and tightness of the quality requirements makes the DVD manufacturing process difficult to set-up.

A system model for the substrate molding process has been previously presented [27]. Consider a process with four process variables (x_j) consisting of cooling time, first stage clamp tonnage, first stage clamp time, and second stage clamp tonnage. The quality attributes (y_i) and specifications are defined as:

$$\begin{bmatrix} -0.30 \\ -50 \\ -0.80 \\ -0.80 \end{bmatrix} < \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \text{Min Tangential Dev} \\ \text{Max Birefringence} \\ \text{Min Radial Deviation} \\ \text{Max Radial Deviation} \end{bmatrix} < \begin{bmatrix} 0.30 \\ 50 \\ 0.80 \\ 0.80 \end{bmatrix} \begin{matrix} \text{deg} \\ nm \\ \text{deg} \\ \text{deg} \end{matrix} \tag{20}$$

Using linear regression techniques, a linear empirical model of the system was generated:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0.345 & -0.021 & -0.009 & 0.058 & -0.041 \\ -19.3 & 16.0 & 0.815 & 10.1 & 0.931 \\ 0.301 & -0.095 & -0.0002 & 0.130 & -0.036 \\ 0.117 & -0.142 & 0.0217 & 0.047 & 0.023 \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{21}$$

The described methods were utilized to define the feasible performance space and subsequent process flexibility index utilizing a normal distribution. The following standard deviations for the quality attributes were observed from the process investigation:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{bmatrix} = \begin{bmatrix} 0.043 \\ 4.0 \\ 0.027 \\ 0.57 \end{bmatrix} \begin{matrix} \text{deg} \\ nm \\ \text{deg} \\ \text{deg} \end{matrix} \tag{22}$$

A molding process was selected to minimize the tangential deviation of the disc substrates while satisfying other quality attributes. The resulting feasibility ranges for the quality attributes are shown by the left set of graphs in Fig. 11. In the figure, the specification ranges on the quality attributes have been uniformly scaled to span the width of the graphs. The inset bars indicate the feasible range of the quality attributes that can be achieved by modifying the processing variables without violating specifications. The vertical lines indicate the current set of quality attributes. The probability of setting the quality attributes within the specified region is 89.4%, while the probability of setting the quality attributes within the feasible region is 85.8%. This results in a process flexibility index, C_f, of 0.86 as defined from Eq. (13).

A process engineer may now wish to quantify the restricted process flexibility when the mean tangential deviation is set to its current value and only the stage one and stage two clamp tonnages are allowed to change. These restrictions on the process greatly reduce the feasible ranges on the other quality attributes as shown in the right hand set of graphs of Fig. 11. In this case, the probability of setting the quality attributes within the specified region remains at 89.4%, while the probability of setting the quality attributes within the feasible region is reduced to 19.7%. This results in a process flexibility index of -0.68, indicative that the process will be unlikely to change other quality attributes while keeping the tangential deviation at 0 and only changing the stage one and stage two clamp tonnages. As such, the process requires additional process variables to impact the quality, or the tangential deviation must be allowed to vary to impact changes in other quality attributes.

5 Discussion

Univariate and multivariate indices have been defined for one and two-sided specifications. The defined process flexibility index has many useful properties for product and process development. For uniformly distributed quality attributes, the process flexibility index is a direct measure of the size of the process feasible space relative to specifications. For probabilistic distributions of quality attributes around the operating point, the process flexibility index is indicative of the likelihood of the process to significantly alter the quality attributes relative to specifications. Indices less than zero indicate that the process is incapable of changing the quality attributes without violating the specification or process control limits. The flexibility index is intended to compare potential design solutions, as such any comparison would assume the specifications (*LSL*, *USL*) are constant. This is of considerable importance since a higher C_f value can be achieved if the specifications are reduced.

The flexibility index is a quantitative measure of the independence of the quality attributes. While similar to Suh's axiomatic design approach [28], the flexibility index operates directly upon the quality attributes and automatically resolves the effect of coupling between multiple quality attributes within the system. The describe methods provide two significant extensions to the axiomatic design approach: 1) solving the feasible set of quality attributes from the axiomatic design matrix, *A*, and 2) assessing an explicit alternative to the information content of system designs. It should also be mentioned that similar metrics can be used for both processing and design variables, as similarly developed for computing the information content and couplings in design [29].

There are two fundamental issues to the broad applicability of the developed methods. First, a probabilistic approach was utilized to estimate the necessity of changes in quality attributes about an operating point. The assumption of normal statistics and independence for changes in quality attributes may not always apply. Frankly, the described approach was developed since it requires no additional information beyond that required for calculating the process capability index. However, the described methods can be extended to nonnormal statistics. Moreover, it is possible to estimate the joint probability by integrating the probability density function across the volume of the *n*-dimensional feasible space.

However, the process flexibility index does not consider the desirability or the cost of changing a quality attribute [30]. Two alternative approaches were considered that could incorporate a utility shape function for each quality attribute [31], or a Taguchi Loss function to estimate the loss in value associated with deviation from the current set of quality attributes [32]. Both these approaches would tend to promote improvements in quality attributes by moving the operating point away from the specification limits. As such, it may be desirable to an asymmetric weighting function to assist the product or process engineer in moving towards a more optimal process.

The second primary limitation is that the algorithms have only been developed for linear system models. Current research has shown that quadratic and nonmonotonic system models result in nonconvex spaces that invalidate the solution of the feasible spaces. The solution of nonconvex polytopes is a challenging yet active field of research [33].

6 Conclusions

Concurrent product and process development relies on specifications for design, analysis, and implementation. During startup and validation, the development team frequently finds that the specifications are incorrect or infeasible, requiring changes in multiple quality attributes, or reductions in cycle time and/or cost. If unexpected, such changes can require significant delay and investment to achieve acceptable product and process performance. The process flexibility index, C_f, assesses the ability of design and processing variables to effect significant changes in the quality attributes. While the process capability index measures the ability of the process to manufacture consistent product, the process flexibility index measures the ability of the process to significantly change the product quality characteristics. Together, the process capability and flexibility indices are useful performance measures that can assist integrated product and process development.

Acknowledgments

This research is supported by National Science Foundation, Division of Design, Manufacture, and Industrial Innovation, grant #9702797 and the Office of Naval Research, grant #N00014-99-0488.

References

[1] Muccio, E. A., 1992, *Plastic Part Technology*, ASM International, Material Park, Ohio, p. 176.
[2] Martin, M. V., 1996, "Design for Variety: A Methodology for Understanding the Costs of Product Proliferation," *Proceedings of the 1996 ASME Design Engineering Technical Conference and Computers in Engineering Conference*, Irvine, CA.
[3] Noaker, P. M., 1996, "CAM's Solid Modeling Connection," Manufacturing Engineering, **117**(5), pp. 361–368.
[4] Blaich, R., 1988, "From Experience-Global Design," Journal of Product Innovation Management, **5**(4), p. 296.
[5] Taninecz, G., 1995, "Faster In, Faster Out," Industry Week, **244**(10), p. 27.
[6] Deitz, D., 1996, "A Re-Engineered Strike Fighter Takes Off," Mech. Eng. (Am. Soc. Mech. Eng.), **118**(11), p. 70.
[7] Hegde, G. G., and Kekre, S., 1992, "Engineering Changes and Time Delays: A Field Investigation," International Journal of Production Economics, **28**(3), pp. 341–352.
[8] Dacey, W. E., 1990, "Cost of Engineering Changes at Various Stages of Electro-Mechanical System Development," *Proceedings of the 4th Design for Manufacturability Conference*.
[9] Kapoor, D., and Kazmer, D., 1997, "The Definition and Use of the Process Flexibility Index," *ASME Design Engineering Technical Conference*, Sacramento, California.
[10] Kazmer, D., Zhu, L., and Hatch, D., "Process Window Derivation with an Application to Optical Media Manufacturing," ASME J. Manuf. Sci. Eng., in press.
[11] Zhu, L., and Kazmer, D., "A Performance-Based Representation for Engineering Design," ASME J. Mech. Des., in press.
[12] Zhu, L., 2001, "A Performance-Based Representation for Engineering Design," *Department of Mechanical and Industrial Engineering*, University of Massachusetts, Amherst, p. 195.
[13] Gross, J., and Yellen, J., 1999, "Graph Theory and Applications," *Discrete Mathematics and Applications*, K. Rosen, ed., New York, CRC Press.
[14] Dantzig, G. B., 1998, *Linear Programming and Extensions*, xvi ed., Princeton University Press, p. 627.
[15] Barber, C. B., Dobkin, D. P., and Huhdanpaa, H., 1996, "The Quickhull Algorithm for Convex Hulls," ACM Trans. Math. Softw., **22**(4), pp. 469–483.
[16] Nazareth, J. L., 1987, *Computer Solutions of Linear Programs*, Oxford University Press, p. 231.
[17] Kane, V. E., 1986, "Process Capability Indices," J. Quality Technol., **18**(1), pp. 41–52.
[18] Feng, C.-X., and Kusiak, A., 1995, "Probabilistic Tolerance Synthesis: A Comparative Study," *Proceedings of the 1995 4th Industrial Engineering Research Conference*, Nashville, TN.
[19] Boyles, R. A., 1991, "The Taguchi Capability Index," J. Quality Technol., **23**(1), p. 17.
[20] Ford, R. B., "Process for the Conceptual Design of Robust Mechanical Systems Going Beyond Parameter Design to Achieve World-Class Quality," *Mechanical Engineering*, Stanford University, Stanford.
[21] Luftig, J. T., and Jordan, V. S., 1998, *Design of Experiments in Quality Engineering*, McGraw-Hill, New York: vii, p. 343.
[22] Kazmer, D. O., and Barkan, P., 1997, "Multi-Cavity Pressure Control in the Filling and Packing Stages of the Injection Molding Process," Polym. Eng. Sci., **37**(11), pp. 1865–1879.
[23] DeGaspari, J., 2000, "All in a Day's Work," Mech. Eng. (Am. Soc. Mech. Eng.), **122**(8), pp. 74–78.
[24] Oshiro, T., Goto, T., and Ishibashi, J., 1997, "Experimental Study of DVD Substrate Quality by Operating Conditions in Injection Molding," *Annual Technical Conference-ANTEC, Conference Proceedings*, Toronto, Canada.
[25] Park, S. J., et al., 1998, "Numerical Analysis of Injection/Compression Molding Process for Center-Gated Disc," *Annual Technical Conference-ANTEC, Conference Proceedings*, Atlanta, GA.
[26] Shin, J. W., Rhee, D. C., and Park, S. J., 1998, "Experimental Study of Optical Disc Birefringence," *Annual Technical Conference-ANTEC, Conference Proceedings*, Atlanta, Ga.
[27] Hatch, D., and Kazmer, D., "Process Transfer Function Development for Optical Media Manufacturing," International Journal of Advanced Manufacturing Technology, in press.
[28] Suh, N. P., 1990, *The Principles of Design*, 1 ed., Oxford Series on Advanced Manufacturing, Oxford University Press, Inc., p. 401.
[29] Frey, D. D., Jahangir, E., and Englehart, F., 2001, "Computing the Information Content of Decoupled Designs," ASME J. Mech. Des., in press.
[30] Roser, C., and Kazmer, D., 2001, "An Economic Design Change Method," ASME J. Mech. Des., in review.
[31] Keeney, R. L., and Raiffa, H., 1993, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, 2 ed., Cambridge University Press, p. 569.
[32] Phadke, M. S., 1989, *Quality Engineering Using Robust Design*, Prentice-Hall, Englewood Cliffs.
[33] Sottile, F., 1997, "Real Enumerative Geometry and Effective Algebraic Equivalence," J. Pure Appl. Algebra, **117–118**(1-3), pp. 601–615.